CHAPTER 2

MOTION IN ONE DIMENSION

Mechanics, the oldest of the physical sciences, is the study of the motion of objects. The calculation of the path of a baseball or of a space probe sent to Mars is among its problems, as is the analysis of the tracks of elementary particles formed following collisions in our largest accelerators. When we describe *motion, we are dealing with the part of mechanics called* kinematics *(from the Greek word for motion, as also in cinema). When we analyze the* causes *of motion we are dealing with* dynamics *(from the Greek word for force, as in dynamite). In this chapter, we deal mostly with kinematics in one dimension. Chapter 3 introduces one-dimensional dynamics, and Chapter 4 extends these concepts to two and three dimensions.*

2-1 KINEMATICS WITH VECTORS

A scouting party has become trapped in a forest away from their field camp (Fig. 2-1). Based on their explorations, the scouts know that they are 2.0 km from the field camp in a direction 30° west of north. They also know that the field camp is located 3.0 km from the base camp in a direction 45° north of east. They wish to radio their position to the base camp so that food and supplies can be dropped by air as close to their position as possible. How can they pinpoint their location relative to the base camp?

Although there are several ways to solve this problem, the most compact way is in terms of *vectors.* Vectors are quantities that have both magnitude and direction and that follow a certain set of mathematical rules for processes such as addition and multiplication. In Fig. 2-1, the position vector $\vec{\mathbf{r}}_1$ (of length 3.0 km in a direction 45° N of E) locates the field camp relative to the base camp. The position vector $\vec{\mathbf{r}}_2$ (of length 2.0 km in a direction 30° W of N) locates the scouting party relative to the field camp.

We wish to find the vector $\vec{\mathbf{r}}$ that locates the scouting party relative to the base camp. Mathematically, we write this relationship as $\vec{\mathbf{r}} = \vec{\mathbf{r}}_1 + \vec{\mathbf{r}}_2$, but the + sign in this equation means something different from its meaning in ordinary arithmetic or algebra. Clearly it does not mean we should add 3.0 km + 2.0 km to find the distance from the base camp to the scouting party; moreover, this equation must also convey some directional information to help locate the scouting party. Note that the equation $\vec{\mathbf{r}} = \vec{\mathbf{r}}_1 + \vec{\mathbf{r}}_2$ does *not* tell us that the distance from the base camp to the scouting party along $\vec{\mathbf{r}}$ is the same as the sum of the distances along $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$. Instead, it tells us

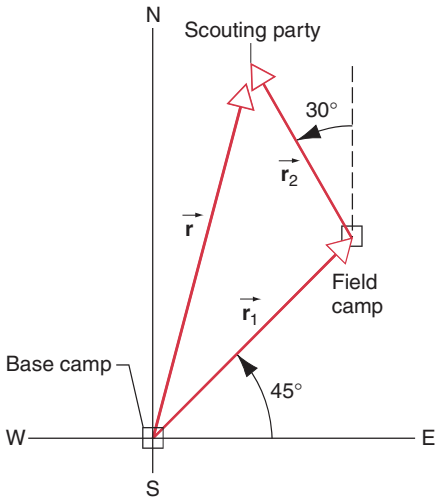

FIGURE 2-1. The relative locations of the base camp, field camp, and scouting party can be specified using vectors.

that we can reach our goal of traveling from the base camp to the scouting party along two equivalent paths, where "equivalent" means that we end up in the same final location.

Position is only one of many quantities in physics that can be represented by vectors. Others include velocity, acceleration, force, momentum, and electromagnetic fields. In contrast to vectors, quantities that can be completely described by specifying only a number (and its units) are called *scalars.* Examples of scalars are mass, time, temperature, and energy.

Kinematics

In this chapter we begin our study of the motion of objects by introducing the terms that are used to describe the motion and showing how they are related to one another. This branch of physics is called *kinematics.* By specifying the *position, velocity,* and *acceleration* of an object, we can describe how the object moves, including the direction of its motion, how that direction changes with time, whether the object speeds up or slows down, and so forth.

For simplicity we will in this chapter consider only the motion of *particles.* By "particle" we often mean a single mass point, such as an electron, but we also can use "particle" to describe an object whose parts all move in exactly the same way. Even a complex object can be treated as a particle if there are no internal motions such as rotations or vibrations of its parts. For example, a rolling wheel cannot be treated as a particle, because a point on its rim moves differently from a point on its axle. (However, a *sliding* wheel *can* be treated as a particle.) Thus an object may be considered a particle for some calculations but not for others. For now we will neglect all internal motions and consider an electron and a freight train on the same basis—as examples of particle motion. Within this approximation, particles can execute a variety of motions: speed up, slow down, even stop and reverse direction or move in curved paths such as circles or parabolas. As long as we can regard these objects as particles, we can use the same set of kinematic equations to describe them.

Many of the laws of physics can be expressed most compactly as relations between quantities expressed as vectors. When a law is written in vector form, it is often easier to understand and to manipulate. Position, velocity, and acceleration, which are the quantities of kinematics, are all vector quantities, and the rules that define them and relate them to one another are vector laws. In this chapter we develop these laws and apply them to motion in a straight line. A more complete demonstration of the usefulness of these vector laws will come in Chapter 4, when we consider two- and three-dimensional motion in curved paths.

In the next section we summarize some of the basic properties of vectors that we will need in kinematics. Further details about the properties of vectors may be found in Appendix H.

2-2 PROPERTIES OF VECTORS

To represent a vector on a diagram we draw an arrow. The length of the arrow is drawn to be proportional to the magnitude of the vector using any convenient scale. Other vectors that are part of the same problem are drawn using the same scale, so that the relative sizes of the arrows are the same as the relative magnitudes of the vectors. (For example, the vector $\vec{\mathbf{r}}_1$ in Fig. 2-1, which represents 3.0 km, is drawn to be 1.5 times as long as the vector $\vec{\mathbf{r}}_2$, which represents 2.0 km.) The direction of the arrow corresponds to the direction of the vector, with the arrowhead giving the sense of the direction. Vectors are represented in this book using boldface type with an arrow, such as $\vec{\mathbf{a}}$ or $\vec{\mathbf{b}}$. In your handwritten work, you can write vectors by placing an arrow above the symbol, such as $\vec{a}$ or $\vec{b}$.

The magnitude or length of a vector is indicated by $|\vec{\mathbf{a}}|$, which gives us no directional information at all about the vector $\vec{\mathbf{a}}$. We will usually write the magnitude of a vector by a single italic symbol such as a, which has the same meaning as $|\vec{\mathbf{a}}|$.

Components of Vectors

We can specify a vector by giving its length and direction, as for example the position vectors of Fig. 2-1. It is often more useful, however, to describe a vector in terms of its *components.* Figure 2-2*a* shows the vector $\vec{\mathbf{a}}$. Its magnitude or length is a and its direction is specified by the angle ϕ, which is measured with respect to the positive x axis. The x and y components of $\vec{\mathbf{a}}$ are defined by

$$a_x = a\cos\phi \qquad \text{and} \qquad a_y = a\sin\phi. \tag{2-1}$$

Although the magnitude a is always positive, the components a_x and a_y may be positive or negative, depending on

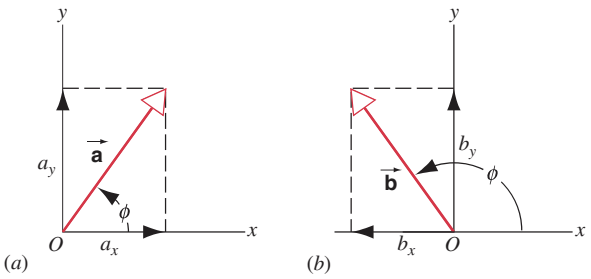

FIGURE 2-2. (*a*) The vector $\vec{\mathbf{a}}$ has component a_x in the x direction and component a_y in the y direction. (*b*) The vector $\vec{\mathbf{b}}$ has a negative x component.

the angle ϕ. For example, as shown in Fig. 2-2*b*, the vector $\vec{\mathbf{b}}$ is located by an angle ϕ that is greater than 90° but less than 180° (ϕ always being measured from the *positive x* axis). For this case, b_x is negative and b_y is positive.

If we know a and ϕ, we can find the components according to Eqs. 2-1. We can also reverse this process—given a_x and a_y, we can find the magnitude of the vector and the angle ϕ by

$$a = \sqrt{a_x^2 + a_y^2} \qquad \text{and} \qquad \tan\phi = a_y/a_x. \qquad (2\text{-}2)$$

The quadrant in which $\vec{\mathbf{a}}$ is located, and thus the value of ϕ, can be determined from the signs of a_x *and* a_y. For example, both −45° (or 315°) and 135° have tangents equal to −1, and thus $a_y/a_x = -1$ for both. For $\phi = -45°$, a_x is positive and a_y is negative, whereas for $\phi = 135°$, a_x is negative and a_y is positive. Knowing the signs of a_x and a_y would allow us to distinguish between these two possibilities. (See Sample Problem 2-3 for another discussion of this problem.)

A more formal way to write a vector in terms of its components is based on a set of *unit vectors.* Unit vectors are vectors of length 1 in the direction of each of the coordinate axes. In the Cartesian coordinate system, the x and y unit vectors are indicated by $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, as shown in Fig. 2-3*a*. Using the unit vectors, we can write the vector $\vec{\mathbf{a}}$ as

$$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}. \qquad (2\text{-}3)$$

Unit vectors are dimensionless; the dimensions of $\vec{\mathbf{a}}$ in Eq. 2-3 are carried by the components a_x and a_y.

The vector relation of Eq. 2-3 is exactly equivalent to the two scalar relations of Eq. 2-1. Sometimes we refer to $a_x\hat{\mathbf{i}}$ and $a_y\hat{\mathbf{j}}$ as the *vector components* of $\vec{\mathbf{a}}$. Figure 2-3*b* shows the vector $\vec{\mathbf{a}}$ and its vector components. Because the physical effect of a vector is identical to the combined physical effects of its vector components, we may occasionally want to analyze problems by replacing a vector by its vector components. Usually when we speak of components, however, we will mean the scalar components of Eq. 2-1.

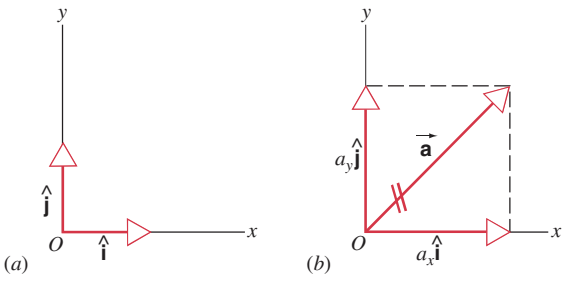

FIGURE 2-3. (*a*) The unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. (*b*) The *vector* components of $\vec{\mathbf{a}}$. When we want to replace $\vec{\mathbf{a}}$ by its vector components, it is helpful to draw a double line through the original vector, as shown; this helps to remind us not to consider the original vector any more.

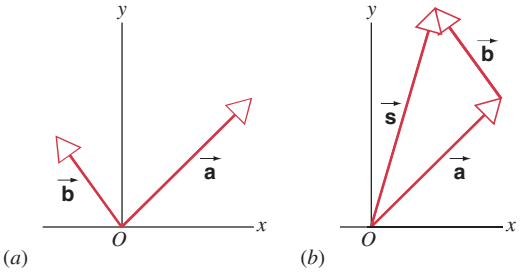

FIGURE 2-4. (*a*) Vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. (*b*) To find the sum $\vec{\mathbf{s}}$ of vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, we slide $\vec{\mathbf{b}}$ without changing its magnitude or direction until its tail is on the head of $\vec{\mathbf{a}}$. Then the vector $\vec{\mathbf{s}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$ is drawn from the tail of $\vec{\mathbf{a}}$ to the head of $\vec{\mathbf{b}}$.

When we write an equation involving vectors, such as $\vec{\mathbf{a}} = \vec{\mathbf{b}}$, we mean that the two vectors are precisely the same: they have the same magnitude and direction. This can occur only if $a_x = b_x$ and $a_y = b_y$. That is,

Two vectors are equal to each other only if their corresponding components are equal.

Adding Vectors

As in the case of Fig. 2-1, we often want to add two or more vectors to find their sum. Consider the two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ in Fig. 2-4*a*. We wish to find the vector $\vec{\mathbf{s}}$ such that $\vec{\mathbf{s}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$.

Figure 2-4*b* shows a graphical construction that allows us to find $\vec{\mathbf{a}} + \vec{\mathbf{b}}$. We first draw the vector $\vec{\mathbf{a}}$. Instead of drawing $\vec{\mathbf{b}}$ with its tail at the origin, as in Fig. 2-4*a*, we move $\vec{\mathbf{b}}$ until its tail coincides with the head of $\vec{\mathbf{a}}$. As long as we don't change its magnitude or direction, we can move a vector in this way. The vector $\vec{\mathbf{s}}$, representing the sum $\vec{\mathbf{a}} + \vec{\mathbf{b}}$, is now drawn from the tail of $\vec{\mathbf{a}}$ to the head of $\vec{\mathbf{b}}$. If we are adding more than two vectors, we can continue placing them tail to head in this way, and the sum vector is drawn from the tail of the first to the head of the last. Often we can use geometric or trigonometric relationships to find the magnitude and direction of the sum vector.

Another way to add vectors is to add their components. That is, $\vec{\mathbf{s}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$ means

$$s_x\hat{\mathbf{i}} + s_y\hat{\mathbf{j}} = (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}) + (b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}})$$
$$= (a_x + b_x)\hat{\mathbf{i}} + (a_y + b_y)\hat{\mathbf{j}}.$$

Setting the x components on the left side of this equation equal to those on the right, and doing the same for the y components, we obtain

$$s_x = a_x + b_x \qquad \text{and} \qquad s_y = a_y + b_y. \qquad (2\text{-}4)$$

To add vectors in this way, we resolve each vector into its components and then add the components (taking into account their algebraic signs) to find the components of the sum vector. Figure 2-5 illustrates this addition. Once we

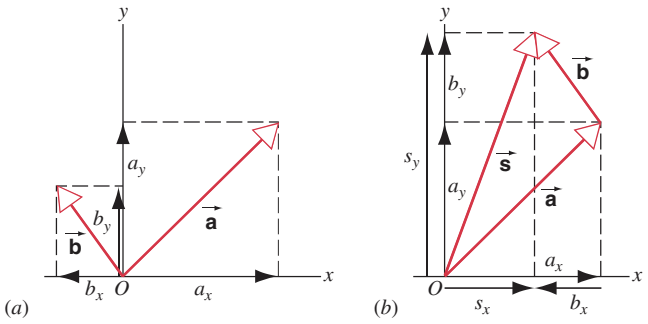

FIGURE 2-5. (*a*) The components of the vectors $\vec{a}$ and $\vec{b}$. (*b*) The sum vector $\vec{s} = \vec{a} + \vec{b}$ can be found by adding the components of $\vec{a}$ to the components of $\vec{b}$. Note that b_x is negative, so that $s_x = a_x + b_x$ involves a subtraction.

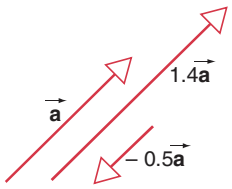

FIGURE 2-6. Multiplication of a vector $\vec{a}$ by a scalar c gives a vector $c\vec{a}$ whose magnitude is c times the magnitude of $\vec{a}$. The vector $c\vec{a}$ has the same direction as $\vec{a}$ if c is positive and the opposite direction if c is negative. Examples are illustrated for $c = +1.4$ and $c = -0.5$.

know the components of the sum vector, we can easily find its magnitude and direction using Eq. 2-2.

Multiplying a Vector by a Scalar

The product of a scalar c and a vector $\vec{a}$, written $c\vec{a}$, is defined to be a new vector whose magnitude is the magnitude of c times the magnitude of $\vec{a}$. Equivalently, the components of this new vector are ca_x and ca_y. The scalar may be a pure number, or it may be a physical quantity with dimensions and units, so that the new vector $c\vec{a}$ represents a physical quantity that is different from $\vec{a}$. To divide a vector by a scalar, as in $\vec{a}/c$, we simply multiply the vector $\vec{a}$ by $1/c$.

Multiplication by a scalar does not change the direction of a vector, except that it may reverse the direction if the scalar is negative. Figure 2-6 shows the effect of multiplying a vector $\vec{a}$ by a positive scalar and a negative scalar.

If we multiply a vector $\vec{b}$ by the scalar -1, we obtain the vector $-\vec{b}$, which is a vector with the same magnitude as $\vec{b}$ but the opposite direction. The components of $-\vec{b}$ are $-b_x$ and $-b_y$. We can use this property to find the difference between two vectors, $\vec{d} = \vec{a} - \vec{b}$, by first writing $\vec{d} = \vec{a} + (-\vec{b})$ and in effect adding the vectors $\vec{a}$ and $-\vec{b}$. Figure 2-7 illustrates the graphical method

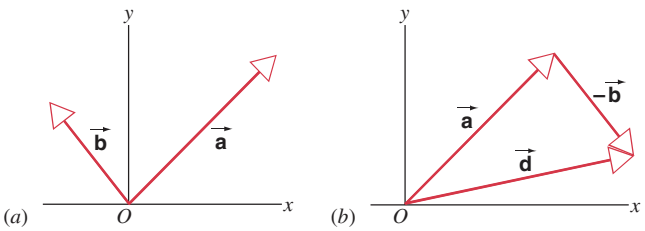

FIGURE 2-7. (*a*) Vectors $\vec{a}$ and $\vec{b}$. (*b*) The difference $\vec{d} = \vec{a} - \vec{b}$ is obtained by adding $-\vec{b}$ to $\vec{a}$.

for adding $\vec{a}$ and $-\vec{b}$ to find $\vec{d} = \vec{a} - \vec{b}$. In analogy with Eq. 2-4, the components of $\vec{d}$ are $d_x = a_x - b_x$ and $d_y = a_y - b_y$.

SAMPLE PROBLEM 2-1. An airplane travels 209 km on a straight course making an angle of 22.5° east of due north. How far north and how far east does the plane travel from its starting point?

Solution We choose the positive x direction to be east and the positive y direction to be north. Next, we draw a displacement vector (Fig. 2-8) from the origin (starting point), making an angle of 22.5° with the y axis (north) inclined along the positive x direction (east). The length of the vector represents a magnitude of 209 km. If we call this vector $\vec{d}$, then d_x gives the distance traveled east of the starting point and d_y gives the distance traveled north of the starting point. We have

$$\phi = 90.0° - 22.5° = 67.5°,$$

so that (see Eqs. 2-1)

$$d_x = d \cos \phi = (209 \text{ km}) (\cos 67.5°) = 80.0 \text{ km},$$

and

$$d_y = d \sin \phi = (209 \text{ km}) (\sin 67.5°) = 193 \text{ km}.$$

In solving this problem, we have assumed that the surface of the Earth can be represented as the xy plane. We know, however,

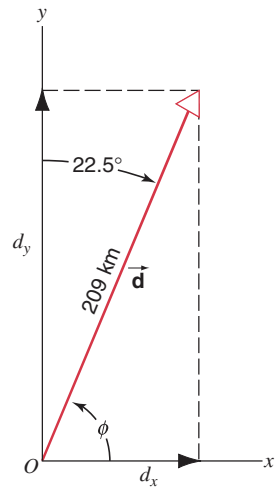

FIGURE 2-8. Sample Problem 2-1.

that the surface of the Earth is not flat but curved, with a radius of about 6400 km. Over small distances, the Earth's surface is approximately flat and we can safely use the xy coordinates. Can you estimate the distance d that the airplane must fly before the use of flat Cartesian coordinates causes a 5% error in the calculated distance that the airplane travels north and east of its starting point?

SAMPLE PROBLEM 2-2. An automobile travels due east on a level road for 32 km. It then turns due north at an intersection and travels 47 km before stopping. Find the vector that indicates the resulting location of the car.

Solution We choose a coordinate system fixed with respect to the Earth, with the positive x direction pointing east and the positive y direction pointing north. The two successive journeys, represented by vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, are then drawn as shown in Fig. 2-9. The resultant $\vec{\mathbf{s}}$ is obtained from $\vec{\mathbf{s}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$. Since $\vec{\mathbf{b}}$ has no x component and $\vec{\mathbf{a}}$ has no y component, using Eq. 2-4 we obtain

$$s_x = a_x + b_x = 32 \text{ km} + 0 = 32 \text{ km},$$

$$s_y = a_y + b_y = 0 + 47 \text{ km} = 47 \text{ km}.$$

The magnitude and direction of $\vec{\mathbf{s}}$ are then (see Eqs. 2-2)

$$s = \sqrt{s_x^2 + s_y^2} = \sqrt{(32 \text{ km})^2 + (47 \text{ km})^2} = 57 \text{ km},$$

$$\tan\phi = \frac{s_y}{s_x} = \frac{47 \text{ km}}{32 \text{ km}} = 1.47, \qquad \phi = \tan^{-1}(1.47) = 56°.$$

The resultant vector $\vec{\mathbf{s}}$ has a magnitude of 57 km and makes an angle of 56° north of east.

SAMPLE PROBLEM 2-3. Three vectors in the xy plane are expressed with respect to the coordinate system as

$$\vec{\mathbf{a}} = 4.3\hat{\mathbf{i}} - 1.7\hat{\mathbf{j}},$$

$$\vec{\mathbf{b}} = -2.9\hat{\mathbf{i}} + 2.2\hat{\mathbf{j}},$$

and

$$\vec{\mathbf{c}} = -3.6\hat{\mathbf{j}},$$

in which the components are given in arbitrary units. Find the vector $\vec{\mathbf{s}}$, which is the sum of these vectors.

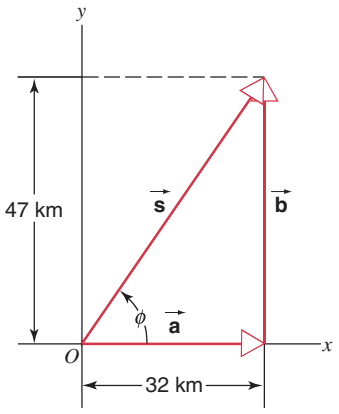

FIGURE 2-9. Sample Problem 2-2.

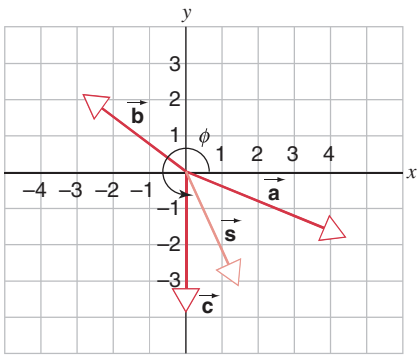

FIGURE 2-10. Sample Problem 2-3.

Solution Generalizing Eqs. 2-4 to the case of three vectors, we have

$$s_x = a_x + b_x + c_x = 4.3 - 2.9 + 0 = 1.4,$$

and

$$s_y = a_y + b_y + c_y = -1.7 + 2.2 - 3.6 = -3.1.$$

Thus

$$\vec{\mathbf{s}} = s_x\hat{\mathbf{i}} + s_y\hat{\mathbf{j}} = 1.4\hat{\mathbf{i}} - 3.1\hat{\mathbf{j}}.$$

Figure 2-10 shows the four vectors. From Eqs. 2-2 we can calculate that the magnitude of $\vec{\mathbf{s}}$ is 3.4 and that the angle ϕ that $\vec{\mathbf{s}}$ makes with the positive x axis, measured counterclockwise from that axis, is

$$\phi = \tan^{-1}(-3.1/1.4) = 294°.$$

Most pocket calculators return angles between +90° and −90° for the arctan. In this case, −66° (which our calculator gives) is equivalent to 294°. However, we would obtain the same angle if we asked for $\tan^{-1}(3.1/-1.4)$, which should give an angle in the second (upper left) quadrant. In applying Eq. 2-2 to find ϕ, the *individual signs* of the components must be considered—it is not sufficient to deal only with the sign of their ratio. Drawing a sketch similar to Fig. 2-10 will keep you from going too far wrong, and if necessary you can convert your calculator's value into a result in the correct quadrant by using the identity $\tan(-\phi) = \tan(180° - \phi)$.

2-3 POSITION, VELOCITY, AND ACCELERATION VECTORS

In kinematics, we describe the motion of a particle using vectors to specify its position, velocity, and acceleration. So far all of our examples have been in two dimensions (the xy plane). Now we consider motion in three dimensions (using an xyz coordinate system).

Figure 2-11 shows a particle moving along an arbitrary path in three dimensions. At any particular time t, the particle can be located by its x, y, and z coordinates, which are the three components of the *position vector* $\vec{\mathbf{r}}$:

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}, \tag{2-5}$$

where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the Cartesian unit vectors, as shown in Fig. 2-11.

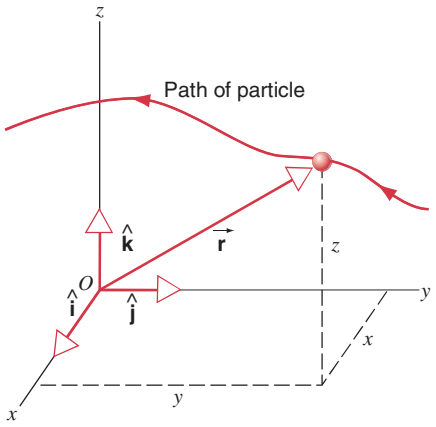

FIGURE 2-11. The position of a particle moving on its path is located by the position vector $\vec{\mathbf{r}}$, which has components x, y, and z. Also shown are the three Cartesian unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$.

Suppose the particle is located at position $\vec{\mathbf{r}}_1$ at time t_1, and it moves along its path to position $\vec{\mathbf{r}}_2$ at time t_2, as shown in Fig. 2-12*a*. We define the *displacement vector* $\Delta\vec{\mathbf{r}}$ as the change in position that occurs in this interval:

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1. \tag{2-6}$$

The three vectors $\vec{\mathbf{r}}_1$, $\Delta\vec{\mathbf{r}}$, and $\vec{\mathbf{r}}_2$ have the same relationship as the vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{s}}$ in Fig. 2-4*b*. That is, using the graphical head-to-tail addition method, $\Delta\vec{\mathbf{r}}$ added to $\vec{\mathbf{r}}_1$ gives the sum $\vec{\mathbf{r}}_2$. Thus $\vec{\mathbf{r}}_2 = \vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}$, which gives Eq. 2-6.

Note from Fig. 2-12*a* that the displacement is not the same as the distance traveled by the particle. The displacement is determined only by the starting and ending points of the interval and not by the path traveled between them.

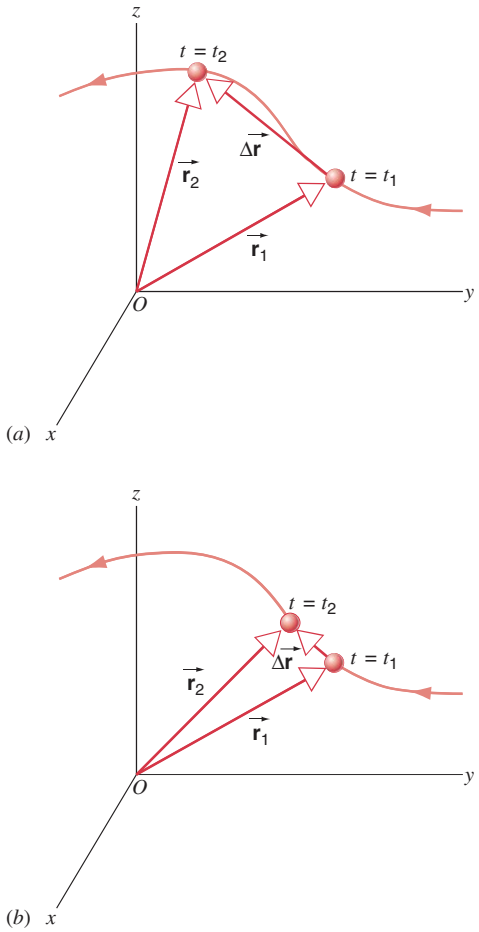

FIGURE 2-12. (*a*) In the interval Δt from t_1 to t_2, the particle moves from position $\vec{\mathbf{r}}_1$ to position $\vec{\mathbf{r}}_2$. Its displacement in that interval is $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$. (*b*) As the interval grows smaller, the displacement vector approaches the actual path of the particle.

Velocity

The *average velocity* in any interval is defined to be the displacement (change in position) divided by the time interval during which the displacement occurs, or

$$\vec{\mathbf{v}}_{\text{av}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}, \tag{2-7}$$

where $\Delta t = t_2 - t_1$. In this equation, the vector $\Delta\vec{\mathbf{r}}$ is multiplied by the positive scalar $1/\Delta t$, *so the vector* $\vec{\mathbf{v}}_{\text{av}}$ *points in the same direction as* $\Delta\vec{\mathbf{r}}$.

Like the displacement, the average velocity in any interval depends only on the locations of the particle at the beginning and end of the interval; it does not depend on whether it speeds up or slows down or even reverses direction in that interval. Note especially that if a particle returns to its starting point, then according to the definition of Eq. 2-7 the average velocity is zero. According to this definition, the average velocity of a race car in the Indianapolis 500 is zero!

Average velocity may be helpful in considering the overall behavior of a particle during some interval, but in describing the details of its motion it would be more useful to have a mathematical function that gives the velocity at every point in the motion. This is the *instantaneous velocity* $\vec{\mathbf{v}}$. When we use the term "velocity," we mean the instantaneous velocity.

To find the instantaneous velocity, we reduce the size of the interval Δt; as we do so, the vector $\Delta\vec{\mathbf{r}}$ approaches the actual path (as in Fig. 2-12*b*), and it becomes tangent to the path in the limit $\Delta t \to 0$. In this case, the average velocity approaches the instantaneous velocity $\vec{\mathbf{v}}$:

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t}. \tag{2-8}$$

The direction of $\vec{\mathbf{v}}$ is tangent to the path of the particle, indicating the direction in which the particle is moving at that instant of time.

Equation 2-8 resembles the definition of a derivative, and we can write it as

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt}. \tag{2-9}$$

The derivative of a vector is found by taking the derivative of each of its components:

$$\frac{d\vec{\mathbf{r}}}{dt} = \frac{d}{dt}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{dz}{dt}\hat{\mathbf{k}}. \tag{2-10}$$

The unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are constant in both magnitude and direction, and so they can be treated as constants in taking the derivative; in other coordinate systems (such as cylindrical or spherical polar systems) the unit vectors may change direction with time and so those vectors do not pass unchanged through the derivative.

The vector $\vec{\mathbf{v}}$ can also be written in terms of its components as

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}. \tag{2-11}$$

Since two vectors can be equal only if their corresponding components are equal, equating Eqs. 2-10 and 2-11 gives three equations:

$$v_x = \frac{dx}{dt}, \qquad v_y = \frac{dy}{dt}, \qquad v_z = \frac{dz}{dt}. \tag{2-12}$$

The three-dimensional vector relation of Eq. 2-9 is equivalent to the three one-dimensional relations of Eq. 2-12.

The term *speed* (taken to mean instantaneous speed) usually refers to the magnitude of the instantaneous velocity, with no indication of direction; that is, the speed v is $|\vec{\mathbf{v}}|$. The speedometer of a car indicates its speed, not its velocity, because it does not specify a direction. Speed is a scalar, since it lacks directional information. We can also define the *average speed:*

$$\text{average speed} = \frac{\text{total distance traveled}}{\text{elapsed time}}. \tag{2-13}$$

It is important to note that the average speed is generally *not* related to the magnitude of the average velocity. For example, the Indianapolis 500 race car with an average velocity of zero (because it starts and ends the race at the same location) certainly does not have an average speed of zero! Equation 2-13 is a scalar equation—the total distance traveled does not provide any information about the direction of the journey.

Both velocity and speed have dimensions of length divided by time, so their SI unit is meters per second (m/s). Other convenient units include miles per hour (mi/h or mph), kilometers per hour (km/h), and so forth.

Acceleration

The velocity of a particle may change in magnitude or direction as it moves. The change in velocity with time is called *acceleration.* In analogy with Eq. 2-7, we can define the *average acceleration* in this interval as the change in velocity per unit time, or

$$\vec{\mathbf{a}}_{\text{av}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}. \tag{2-14}$$

The change in velocity $\Delta\vec{\mathbf{v}}$ means $\vec{\mathbf{v}}_{\text{final}} - \vec{\mathbf{v}}_{\text{initial}}$. As in the case of average velocity, the average acceleration tells us nothing about the variation of $\vec{\mathbf{v}}$ during the interval Δt. The direction of $\vec{\mathbf{a}}_{\text{av}}$ is the same as the direction of $\Delta\vec{\mathbf{v}}$.

The *instantaneous acceleration* $\vec{\mathbf{a}}$ is obtained from the limit of Eq. 2-14 for vanishingly small time intervals:

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t}. \tag{2-15}$$

Once again this can be expressed as a derivative:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} \tag{2-16}$$

and by analogy with Eqs. 2-10 and 2-11 we can write the components of the instantaneous acceleration vector as

$$a_x = \frac{dv_x}{dt}, \qquad a_y = \frac{dv_y}{dt}, \qquad a_z = \frac{dv_z}{dt}. \tag{2-17}$$

Note that vector equations such as Eq. 2-16 can simplify notation, and they also remind us how to separate the components (for example, a_x has no effect on v_y or v_z).

In general the direction of the acceleration has no relation to the direction of $\vec{\mathbf{v}}$. It is possible for $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ to be parallel, antiparallel, or perpendicular to each other, or at any other relative angle.

Because $\vec{\mathbf{v}}$ is a vector quantity, a change in its direction gives an acceleration, even if its magnitude is unchanged. Thus motion at constant speed can be accelerated motion. For example, the components of $\vec{\mathbf{v}}$ can change in such a way that the magnitude of $\vec{\mathbf{v}}$ $(= \sqrt{v_x^2 + v_y^2 + v_z^2})$ remains constant. This situation is found most commonly in uniform circular motion, which we discuss in Section 4-5.

Because acceleration is defined as velocity divided by time, its dimensions are length/time^2. In the SI system, the units of acceleration are m/s^2. Sometimes you may see this written as m/s/s, which is read as "meters per second per second." This emphasizes that acceleration is change in velocity per unit time.

SAMPLE PROBLEM 2-4. A particle moves in the xy plane so that its x and y coordinates vary with time according to $x(t) = At^3 + Bt$ and $y(t) = Ct^2 + D$, where $A = 1.00$ m/s^3, $B = -32.0$ m/s, $C = 5.0$ m/s^2, and $D = 12.0$ m. Find the position, velocity, and acceleration of the particle when $t = 3$ s.

Solution The position is given by Eq. 2-5, with the expressions given for $x(t)$ and $y(t)$:

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = (At^3 + Bt)\hat{\mathbf{i}} + (Ct^2 + D)\hat{\mathbf{j}}.$$

Evaluating this expression at $t = 3$ s, we obtain

$$\vec{\mathbf{r}} = (-69\text{ m})\hat{\mathbf{i}} + (57\text{ m})\hat{\mathbf{j}}.$$

The velocity components are found from Eq. 2-12:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(At^3 + Bt) = 3At^2 + B$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(Ct^2 + D) = 2Ct.$$

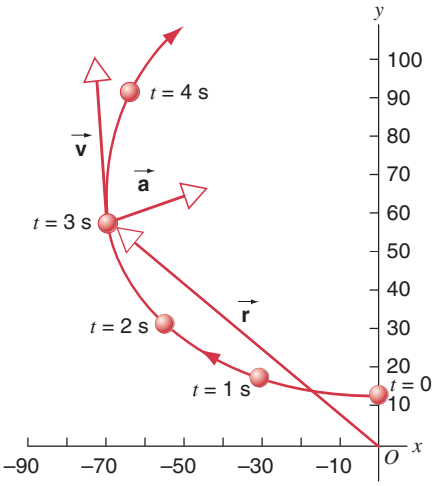

FIGURE 2-13. Sample Problem 2-4. The path of the moving particle is shown, and its positions at $t = 0$, 1, 2, 3, and 4 s are indicated. At $t = 3$ s, the vectors representing its position, velocity, and acceleration are shown. Note that there is no particular relationship between the directions of $\vec{\mathbf{r}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{a}}$ or the lengths of the vectors representing them.

Evaluating the components at $t = 3$ s and using Eq. 2-11, we obtain

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (-5\text{ m/s})\hat{\mathbf{i}} + (30\text{ m/s})\hat{\mathbf{j}}.$$

The components of the acceleration are

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(3At^2 + B) = 6At$$

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(2Ct) = 2C.$$

At $t = 3$ s, the acceleration is

$$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = (18\text{ m/s}^2)\hat{\mathbf{i}} + (10\text{ m/s}^2)\hat{\mathbf{j}}.$$

Figure 2-13 shows the path of the particle from $t = 0$ to $t = 4$ s. The position, velocity, and acceleration vectors at $t = 3$ s are drawn. (Because $\vec{\mathbf{r}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{a}}$ are expressed in different units, the lengths of the arrows drawn in Fig. 2-13 have no relationship to one another.) The vector $\vec{\mathbf{r}}$ locates the particle relative to the origin. The vector $\vec{\mathbf{v}}$ is tangent to the path of the particle, as shown at $t = 3$ s. The acceleration $\vec{\mathbf{a}}$ represents the change in velocity, and the direction of $\vec{\mathbf{a}}$ at $t = 3$ s is reasonable based on how the direction of $\vec{\mathbf{v}}$ changes in the interval around $t = 3$ s.

2-4 ONE-DIMENSIONAL KINEMATICS

Now that we have established the definitions of the important quantities for describing motion, let's look at some examples of how they can be applied. For simplicity, we consider motion only in one dimension, so we will use only one component of Eqs. 2-5, 2-12, and 2-17.

In one-dimensional kinematics, a particle can move only along a straight line. It may change its speed or even reverse direction, but its motion is always along the line. Within this limitation, we can consider many different physical situations, such as a falling stone, an accelerating train, a braking car, a sliding hockey puck, a crate being pulled up a ramp, or a fast-moving electron in an x-ray tube.

We can describe the motion of a particle in two ways: with mathematical equations and with graphs. Either way gives information about the problem, and often we will want to use both methods. The mathematical approach is usually better for solving problems, because it permits greater precision than the graphical method. On the other hand, the graphical method often provides more physical insight than a set of equations.

Here are some possible kinds of motion with the equations or graphs that describe them.

1. *No motion at all.* Here the particle occupies the same position at all times. Suppose the particle is on the x axis at the coordinate A, so that (at all times)

$$x(t) = A. \tag{2-18}$$

A graph of this "motion" is shown in Fig. 2-14*a*. The velocity, which remains constant at zero, is shown in Fig. 2-14*b*. The physical situation represented here might be, for example, a bead that can slide freely along a straight wire. In this case the bead is at rest at $x = A$.

Often in problems in kinematics we want to know how the position and velocity depend on the time as a particle moves. For this reason we write the position coordinate here as a function of the time as $x(t)$. For the same reason we plot the graph in Fig. 2-14 with x as the dependent variable (on the vertical axis) and t as the independent variable (on the horizontal axis). Placing x on the vertical axis does *not* mean that the particle moves vertically; in this situation, the wire on which the bead slides can have any direction.

2. *Motion at constant velocity.* For motion in one dimension (which we choose to be the x direction), the veloc-

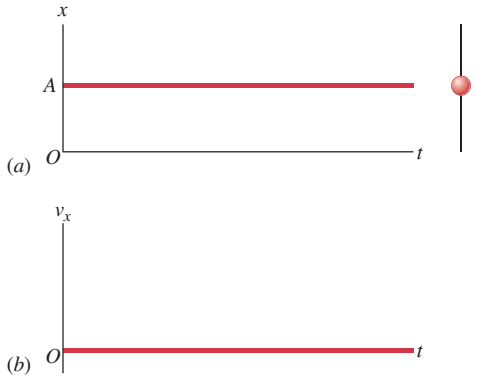

FIGURE 2-14. (*a*) The position and (*b*) the velocity of a bead on a wire at rest $x = A$.

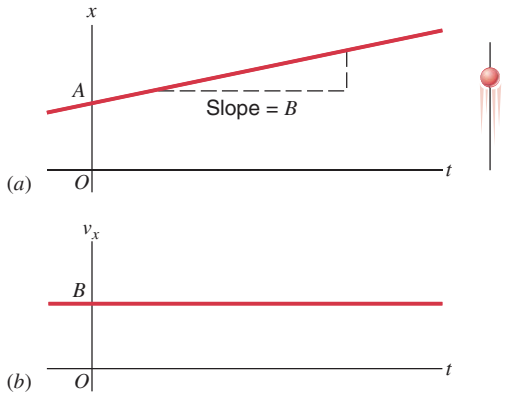

FIGURE 2-15. (*a*) The position and (*b*) the velocity of a bead sliding in one dimension along a wire with constant velocity. The velocity is equal to the slope B of the graph of $x(t)$. The graph of $v_x(t)$ is the horizontal line $v_x = B$.

ity v_x can be positive, if the particle is moving in the direction of increasing x, or negative, if it is moving in the opposite direction. When the velocity is constant, the graph of position against time is a straight line. As Eq. 2-12 ($v_x = dx/dt$) shows, the rate of change of the position is the velocity. In the graph of x against t, the rate of change is the slope of the graph; the greater the velocity, the greater the slope. Figure 2-15*a* shows this graph, whose mathematical form can be expressed as

$$x(t) = A + Bt, \tag{2-19}$$

which is in the customary form as the equation of a straight line (more commonly expressed as $y = mx + b$) with slope B. Note that $v_x = dx/dt = B$; Fig. 2-15*b* shows the constant velocity.

As shown by Fig. 2-15 and also by Eq. 2-19, the particle is at the position $x = A$ when $t = 0$. It is moving in the direction of increasing x, so the slope B (and correspondingly the velocity v_x) is positive.

3. *Accelerated motion.* With acceleration defined as the rate of change of the velocity, accelerated motion then corresponds to motion in which the velocity changes. Since the velocity is the slope of the graph of $x(t)$, the slope must change in accelerated motion. These graphs are therefore curves rather than straight lines. Two examples of accelerated motion are:

$$x(t) = A + Bt + Ct^2, \tag{2-20}$$

$$x(t) = D\cos\omega t. \tag{2-21}$$

In the first case (Fig. 2-16*a*), assuming C is positive, the slope is increasing as the particle moves, corresponding to an increase in the positive velocity of the particle. According to Eqs. 2-12 and 2-17, $a_x = dv_x/dt = d^2x/dt^2$. For Eq. 2-20, $d^2x/dt^2 = 2C$, so the acceleration is constant. In the second case (Fig. 2-16*b*), the particle oscillates between $x = +D$ and $x = -D$; its velocity changes from positive to negative as the derivative of Eq. 2-21 changes sign.

Often the complete descriptions of motion are more complicated than these simple examples. Here are some additional cases to consider:

4. *Accelerating and braking car.* A car is at rest for a time and then begins accelerating until it reaches a certain velocity. It then moves for a time at that velocity, after which the brakes are applied to bring the car to rest again. Figure 2-17*a* shows the acceleration in the various time intervals; for simplicity we assume the acceleration is constant during the intervals when the car is speeding up and slowing down. The acceleration is zero when the car is at rest or when it is moving with constant velocity (as suggested by Eq. 2-17, when v_x is a constant then $a_x = dv_x/dt = 0$).

Figure 2-17*b* shows the velocity in each interval. Where $a_x = 0$, v_x is constant. Where a_x is a positive or negative constant, v_x is represented by a straight line with a corresponding positive or negative slope. At any point, the value of a_x can be found from the slope of the graph of v_x against t.

In a similar way we can obtain the position versus time graph $x(t)$, which is shown in Fig. 2-17*c*. Note that the slope of $x(t)$ gives $v_x(t)$, as required by Eq. 2-12 ($v_x = dx/dt$). For example, in the interval when the car is speeding up, the slope of $x(t)$ is gradually increasing, corresponding exactly to the increase in velocity. In the interval when the car is moving at constant velocity, $x(t)$ is represented by a straight line with a constant slope.

These graphs are somewhat idealized; a real car cannot go instantly from a state of rest to a state of constant acceleration. In practice, the sudden jumps in $a_x(t)$ would instead be connected by a continuous curve, and the sharp bends in the graph of $v_x(t)$ would become rounded.

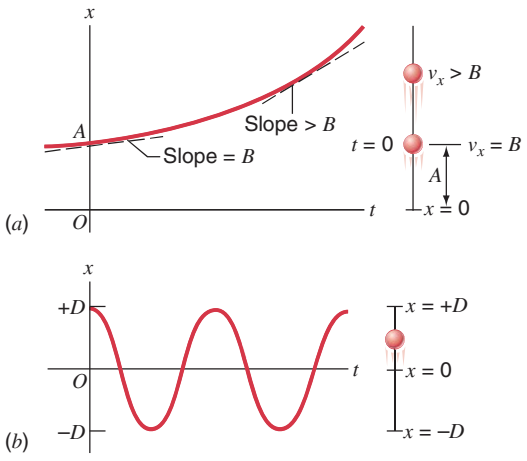

FIGURE 2-16. (*a*) A bead sliding along a wire in one dimension moves in the positive x direction with ever increasing velocity. The velocity is equal to the slope of the curve describing the particle's motion; you can see how the slope of the curve continually increases. (*b*) A bead sliding along a wire in one dimension oscillates between $x = +D$ and $x = -D$.

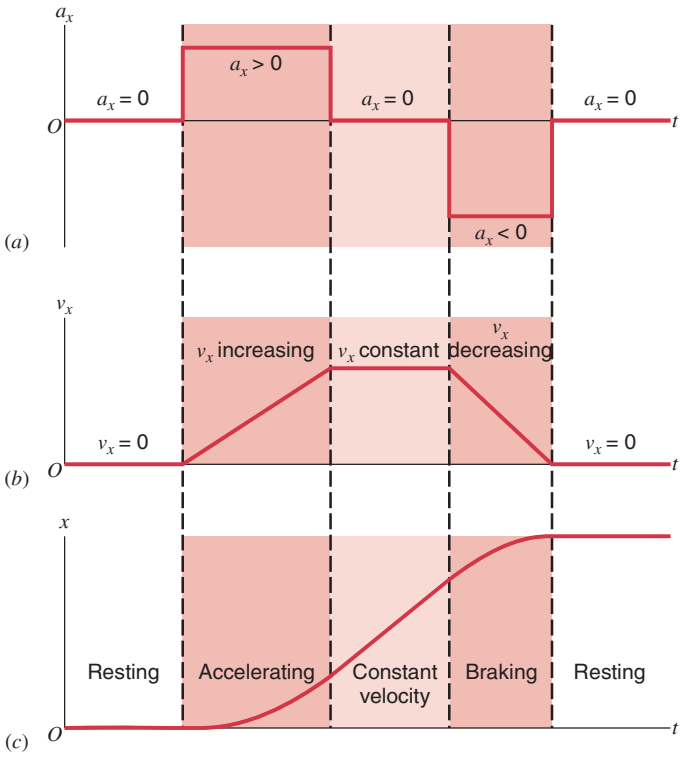

FIGURE 2-17. (*a*) The acceleration, (*b*) the velocity, and (*c*) the position of a car that starts at rest, accelerates for an interval, then moves with constant velocity, and then brakes with negative acceleration to rest again. In reality, we cannot instantly change the acceleration of a car from one value to another; both $a_x(t)$ and $v_x(t)$ for a real car would be smooth and continuous. Smooth curves would connect the flat $a_x(t)$ segments, and the sharp bends in $v_x(t)$ would become rounded.

5. *A falling object.* As we will discuss later in this chapter, when an object falls near the Earth's surface, it experiences a constant downward acceleration due to gravity. In this problem we take the y axis to be vertical and choose the upward direction as positive, so that the acceleration has a negative y component a_y. Figure 2-18*a* shows the acceleration $a_y(t)$ equal to a constant negative value.

If an object is released from rest, its downward (negative) velocity increases in magnitude due to this acceleration. Since a_y is a constant, $v_y(t)$ must be a straight line whose (negative) slope is equal to a_y, as shown in Fig. 2-18*b*. The vertical coordinate y begins at some positive value corresponding to the height H at which the object was originally released, and $y(t)$ decreases gradually to zero as the object falls to the ground, as shown in Fig. 2-18*c*. Initially the slope of $y(t)$ is zero, because $v_y(t)$ is initially zero. As v_y becomes more negative, the corresponding slope of $y(t)$ becomes more negative, as you can see by drawing lines tangent to $y(t)$ at various locations.

6. *A rebounding ball bearing.* Consider now a small steel ball bearing dropped from rest onto a hard surface from which it rebounds. We again choose the vertical direction as the y axis for this problem, and we take the upward direction to be positive.

After the ball bearing strikes the surface, it rebounds upward. We assume that the velocity simply changes direction upon contact with the surface—its magnitude remains unchanged. (In reality, there is a small loss of speed because the ball bearing and the surface are not perfectly elastic.)

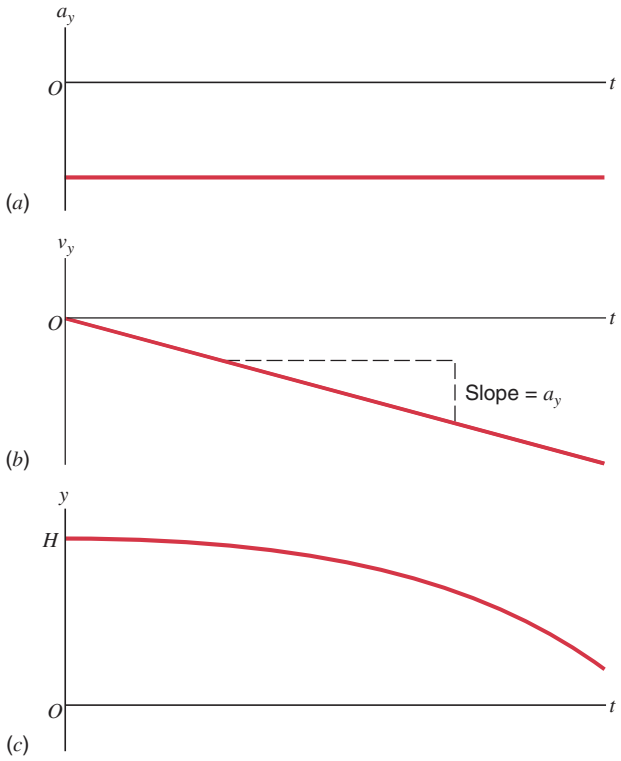

FIGURE 2-18. (*a*) The acceleration, (*b*) velocity, and (*c*) position of an object dropped from rest and accelerated downward by the Earth's gravity. The acceleration is a negative constant, which is equal to the slope of $v_y(t)$.

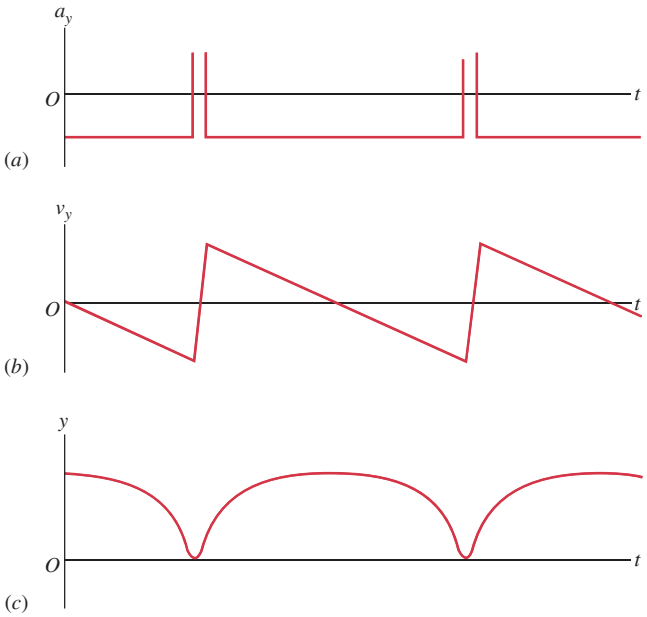

FIGURE 2-19. (*a*) The acceleration, (*b*) velocity, and (*c*) position of a ball bearing that falls and rebounds from a hard surface. The sudden jumps in $v_y(t)$, corresponding to the short intervals when the ball bearing is in contact with the surface, have been drawn as if the acceleration is equal to a large positive constant in those intervals. Note that $y(t)$ reaches its maximum value and that the tangent to $y(t)$ is horizontal when $v_y(t)$ goes through zero.

During the very short interval of time that the ball bearing is in contact with the surface, a large upward (positive) acceleration must act to reverse the direction of its velocity. Note that an acceleration is present even though the magnitude of the velocity is unchanged; an acceleration must occur whenever the magnitude *or the direction* of a velocity is changed.

As the ball bearing rises after the bounce, there is again a (constant) downward acceleration due to gravity, which eventually brings the ball bearing to rest for an instant, after which it begins falling toward the surface again.

Figure 2-19 shows the acceleration, velocity, and position for the ball bearing. Once, again $a_y(t)$ corresponds to the slope of $v_y(t)$, and $v_y(t)$ corresponds to the slope of $y(t)$.

This is also a somewhat idealized representation of this motion. The exact behavior during the instant of collision may be very complicated and will certainly not be characterized by a constant acceleration, as we have assumed here. Nevertheless, the overall behavior should look very similar to Fig. 2-19.

Equations of One-Dimensional Kinematics

We can directly apply the vector equations of Section 2-3 to motion in one dimension, which we take to be the x direction. Let the particle start at location x_1 at time t_1 and move to location x_2 at time t_2. In the time interval $\Delta t = t_2 - t_1$, the displacement of the particle is $\Delta x = x_2 - x_1$. According to Eq. 2-7, its average velocity is

$$v_{\text{av},x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \tag{2-22}$$

The average velocity tells us about the average behavior during the time interval Δt. It depends only on the initial and final locations of the particle and not on the path of the particle between x_1 and x_2. If we assume that Δt is positive (that is, our clocks are running forward), then according to Eq. 2-22 and the rule for multiplying a vector by a scalar, the sign of $v_{\text{av},x}$ is determined by the sign of Δx. If $v_{\text{av},x} > 0$, then on the average the particle moves from a smaller x coordinate to a larger x coordinate (that is, it moves in the positive x direction). For example, the particle might move from $x_1 = -8$ m to $x_2 = -4$ m, or from $x_1 = -3$ m to $x_2 = +1$ m, or from $x_1 = +2$ m to $x_2 = +6$ m. In each case, $\Delta x = +4$ m, and so $v_{\text{av},x} > 0$. If $v_{\text{av},x} < 0$, the particle on the average moves in the negative x direction, for example from $x_1 = +5$ m to $x_2 = +2$ m or from $x_1 = -3$ m to $x_2 = -6$ m (both of which have $\Delta x = -3$ m).

The instantaneous velocity follows from Eq. 2-12:

$$v_x = \frac{dx}{dt}. \tag{2-23}$$

Equation 2-23 allows us to analyze the examples of one-dimensional motion discussed previously in this section. For example, for the motion at constant velocity shown in Fig. 2-15*a*, taking $x(t) = A + Bt$ gives $v_x = dx/dt = B$, as shown in Fig. 2-15*b*.

In the case of accelerated motion in which the particle is moving with velocity v_{1x} at time t_1 and accelerates to velocity v_{2x} at time t_2, Eq. 2-14 gives

$$a_{\text{av},x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}. \tag{2-24}$$

The sign of the average acceleration is determined by the sign of Δv_x; for example, a change in velocity from $v_{1x} = -9$ m/s to $v_{2x} = -4$ m/s or from $v_{1x} = +4$ m/s to $v_{2x} = +9$ m/s both correspond to $\Delta v_x = +5$ m/s and to positive $a_{\text{av},x}$, whereas a change from $v_{1x} = +9$ m/s to $v_{2x} = +4$ m/s or from $v_{1x} = -4$ m/s to $v_{2x} = -9$ m/s both give $\Delta v_x = -5$ m/s and negative $a_{\text{av},x}$.

As in the case of average velocity, the average acceleration depends only on the difference between the starting and ending velocities in the interval and not on the detailed motion during the interval. All motions that result in the same Δv_x in the interval Δt will give the same average acceleration.

The instantaneous acceleration is found in Eq. 2-17:

$$a_x = \frac{dv_x}{dt}. \tag{2-25}$$

For example, in the case shown in Fig. 2-16*a* with $x(t) = A + Bt + Ct^2$, Eq. 2-23 gives $v_x = B + 2Ct$ and Eq. 2-25 gives $a_x = 2C$.

Using Eqs. 2-23 and 2-25, you should review the motions graphed in Figs. 2-17, 2-18, and 2-19 to be sure you understand how position, velocity, and acceleration are related. Note especially that the acceleration is the slope of the graph of $v(t)$. Compare Figs. 2-17*a* and *b* to see that where the slope of $v_x(t)$ is zero (the horizontal line segments) then $a_x = 0$; where $v_x(t)$ is increasing (line segment with positive slope), a_x is a positive constant.

The acceleration may be positive or negative independent of whether the velocity is positive or negative. For example, let's consider an elevator moving vertically; we'll call that the y direction and take up as positive. If the elevator is moving upward but slowing down, v_y is positive but a_y is negative; in this case the velocity vector points in the $+y$ direction and the acceleration vector in the $-y$ direction. If the elevator is moving downward and braking, then v_y is negative but a_y is positive. These two cases, in which the velocity and acceleration vectors have opposite directions so that the speed (the magnitude of the velocity) is decreasing, are often referred to as *deceleration.*

The Limiting Process

It is interesting to observe how the average velocity approaches the instantaneous velocity as Δt approaches zero. As an example, we consider the case $x(t) = 5.0 + 1.0t + 6.0t^2$, where x is in meters and t is in seconds. We arbitrarily select the point $x_1 = 12$ m, $t_1 = 1.0$ s and calculate the value of $v_{av,x}$ using Eq. 2-22 by choosing a series of points x_2, t_2 that approach closer and closer to x_1, t_1 to simulate the limiting process. The calculated values are listed in Table 2-1. Note that they seem to be approaching the value $v_{av,x} = 13$ m/s. To compare this limiting value of $v_{av,x}$ with the instantaneous value, we use Eq. 2-23 to find $v_x(t) = 1.0 + 12.0t$, which does indeed evaluate to $v_x = 13$ m/s when $t = 1.0$ s.

Figure 2-20 gives a graphical illustration of this limiting process. As point 2 moves closer to point 1, the line connecting the two points, whose slope represents the average velocity in that interval, approaches the tangent at point 1. This indicates that the average velocity approaches the instantaneous velocity in this limit, as our mathematical calculation showed.

TABLE 2-1 The Limiting Process

Initial Point		*Final Point*		*Average Velocity*
x_1 (m)	t_1 (s)	x_2 (m)	t_2 (s)	$\Delta x/\Delta t$ (m/s)
12	1.0	62	3.0	25
12	1.0	45	2.5	22
12	1.0	31	2.0	19
12	1.0	20	1.5	16
12	1.0	13.4	1.1	13.6
12	1.0	12.7	1.05	13.3
12	1.0	12.1	1.01	13.06

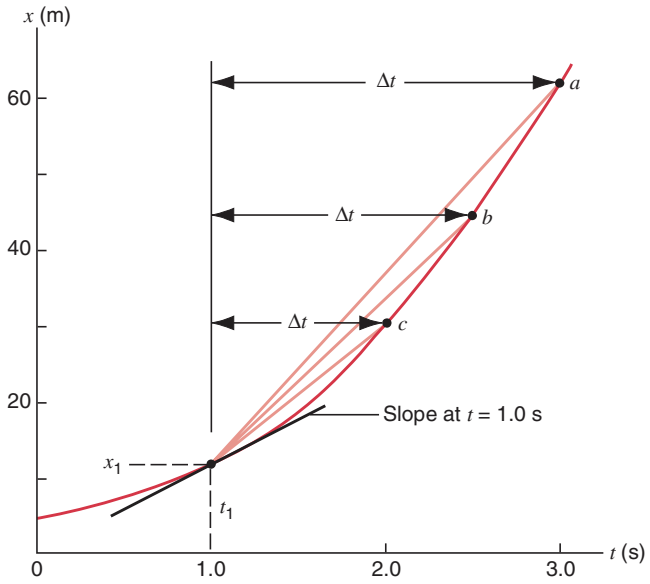

FIGURE 2-20. The interval Δt grows smaller, in this case as we keep t_1 fixed and move the other endpoint t_2 closer to t_1. In the limit, the interval goes to zero and the chord becomes a tangent. The three values of x_2 and t_2 shown here, labeled *a*, *b*, and *c*, correspond to the first three lines of Table 2-1.

SAMPLE PROBLEM 2-5. You drive your BMW down a straight road for 5.2 mi at 43 mi/h, at which point you run out of gas. You walk 1.2 mi farther, to the nearest gas station, in 27 min. What is your average velocity from the time that you start your car to the time that you arrive at the gas station?

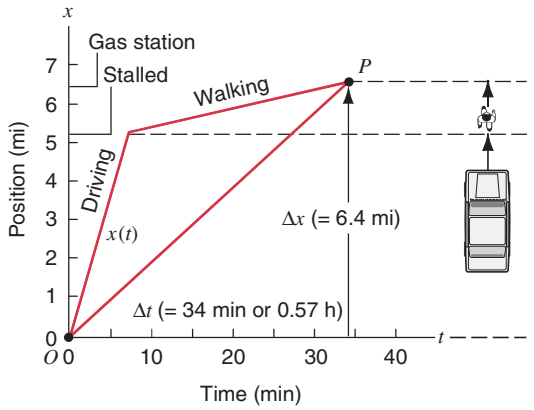

FIGURE 2-21. Sample Problem 2-5. The lines marked "Driving" and "Walking" show motions at different constant velocities for the two portions of the trip. The average velocity is the slope of the line *OP*.

Solution You can find your average velocity from Eq. 2-22 if you know both Δx, the net displacement, and Δt, the corresponding elapsed time. These quantities are

$$\Delta x = 5.2 \text{ mi} + 1.2 \text{ mi} = 6.4 \text{ mi}$$

and

$$\Delta t = \frac{5.2 \text{ mi}}{43 \text{ mi/h}} + 27 \text{ min}$$

$$= 7.3 \text{ min} + 27 \text{ min} = 34 \text{ min} = 0.57 \text{ h}.$$

From Eq. 2-22 we then have

$$v_{\text{av},x} = \frac{\Delta x}{\Delta t} = \frac{6.4 \text{ mi}}{0.57 \text{ h}} = 11.2 \text{ mi/h}.$$

The $x(t)$ plot of Fig. 2-21 helps us to visualize the problem. Points O and P define the interval for which we want to find the average velocity, this quantity being the slope of the straight line connecting these points.

SAMPLE PROBLEM 2-6. Figure 2-22*a* shows six successive "snapshots" of a particle moving along the x axis. At $t = 0$ it is at position $x = +1.00$ m to the right of the origin; at $t = 2.5$ s it has come to rest at $x = +5.00$ m; at $t = 4.0$ s it has returned to $x = 1.4$ m. Figure 2-22*b* is a plot of position x versus time t for this motion, and Figs. 2-22*c* and 2-22*d* show the corresponding velocity and acceleration of the particle. (*a*) Find the average velocity for the intervals AD and DF. (*b*) Estimate the slope of $x(t)$ at points B and F and compare with the corresponding points on the $v_x(t)$ curve. (*c*) Find the average acceleration in the intervals AD and AF. (*d*) Estimate the slope of $v_x(t)$ at point D and compare with the corresponding value of $a_x(t)$.

Solution (*a*) From Eq. 2-22,

$$AD: \quad v_{\text{av},x} = \frac{\Delta x_{AD}}{\Delta t_{AD}} = \frac{x_D - x_A}{t_D - t_A}$$

$$= \frac{5.0 \text{ m} - 1.0 \text{ m}}{2.5 \text{ s} - 0.0 \text{ s}} = +1.6 \text{ m/s}$$

$$DF: \quad v_{\text{av},x} = \frac{\Delta x_{DF}}{\Delta t_{DF}} = \frac{x_F - x_D}{t_F - t_D}$$

$$= \frac{1.4 \text{ m} - 5.0 \text{ m}}{4.0 \text{ s} - 2.5 \text{ s}} = -2.4 \text{ m/s}$$

In the interval AD, the positive sign for $v_{\text{av},x}$ tells us that, on the average, the particle moves in the direction of increasing x (that is, to the right in Fig. 2-22*a*) during that interval. The negative sign for $v_{\text{av},x}$ in the interval DF tells us that the particle is, on the average, moving in the direction of decreasing x (to the left in Fig. 2-22*a*) during that interval.

(*b*) From the tangents to $x(t)$ drawn at points B and F in Fig. 2-22*b* we estimate the following:

$$\text{point } B: \quad \text{slope} = \frac{4.5 \text{ m} - 2.8 \text{ m}}{1.5 \text{ s} - 0.5 \text{ s}} = +1.7 \text{ m/s}$$

$$\text{point } F: \quad \text{slope} = \frac{1.4 \text{ m} - 4.5 \text{ m}}{4.0 \text{ s} - 3.5 \text{ s}} = -6.2 \text{ m/s}$$

From $v_x(t)$ in Fig. 2-22*c* we estimate $v_x = +1.7$ m/s at point B and $v_x = -6.2$ m/s at part F, in agreement with the slopes of $x(t)$. As expected, $v_x(t) = dx/dt$.

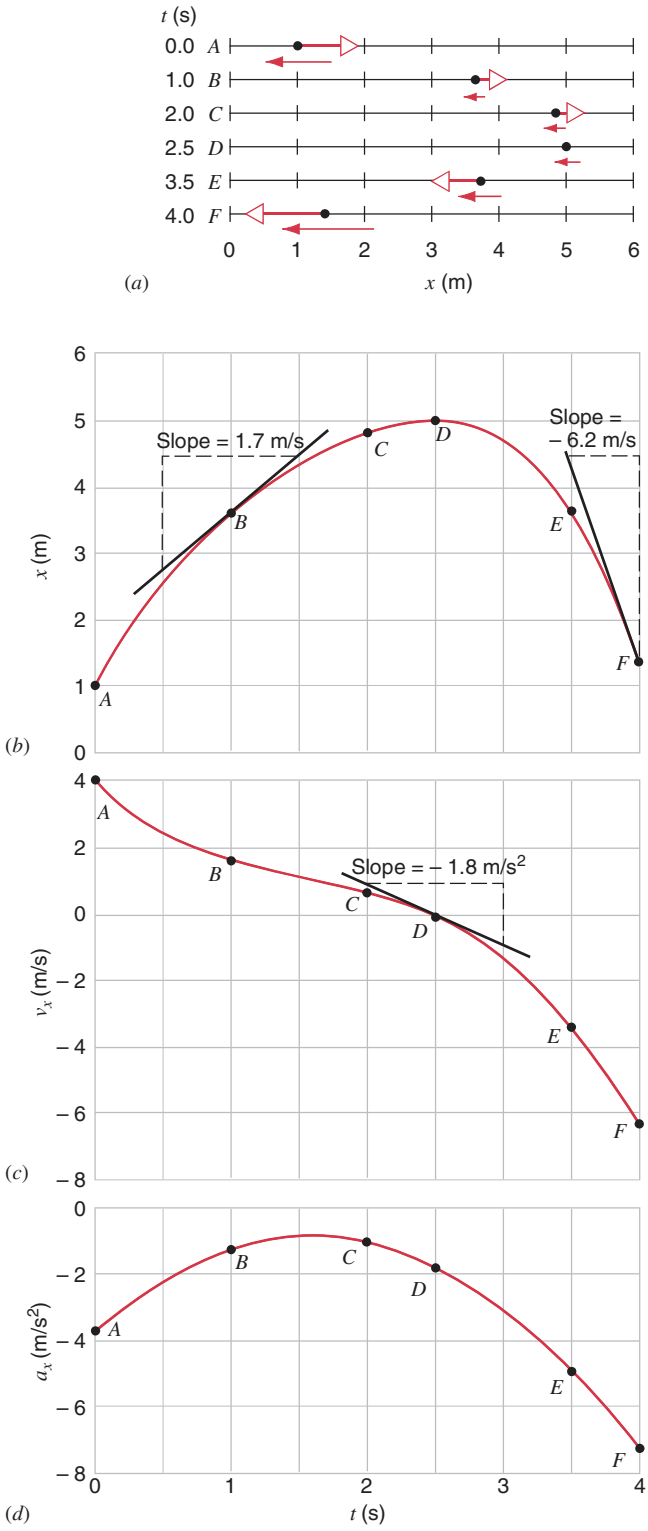

FIGURE 2-22. Sample Problem 2-6. (*a*) Six consecutive "snapshots" of a particle moving along the x axis. The arrow *through* the particle shows its instantaneous velocity, and the arrow *below* the particle shows its instantaneous acceleration. (*b*) A plot of $x(t)$ for the motion of the particle. The six points A–F correspond to the six snapshots. (*c*) A plot of $v_x(t)$. (*d*) A plot of $a_x(t)$.

(*c*) From Eq. 2-24,

$$AD: \quad a_{\text{av},x} = \frac{\Delta v_{AD}}{\Delta t_{AD}} = \frac{v_D - v_A}{t_D - t_A}$$

$$= \frac{0.0 \text{ m/s} - 4.0 \text{ m/s}}{2.5 \text{ s} - 0.0 \text{ s}} = -1.6 \text{ m/s}^2$$

$$AF: \quad a_{\text{av},x} = \frac{\Delta v_{AF}}{\Delta t_{AF}} = \frac{v_F - v_A}{t_F - t_A}$$

$$= \frac{-6.2 \text{ m/s} - 4.0 \text{ m/s}}{4.0 \text{ s} - 0.0 \text{ s}} = -2.6 \text{ m/s}^2$$

(*d*) From the line drawn tangent to $v_x(t)$ at D, we estimate the following:

$$\text{slope} = \frac{-0.9 \text{ m/s} - 0.9 \text{ m/s}}{3.0 \text{ s} - 2.0 \text{ s}} = -1.8 \text{ m/s}^2.$$

At point D on the $a_x(t)$ graph we see $a_x = -1.8 \text{ m/s}^2$. Thus $a_x = dv_x/dt$. Examining the $v_x(t)$ graph of Fig. 2-22*c*, we see that its slope is negative at all times covered by the graph, and thus $a_x(t)$ should be negative. Figure 2-22*d* bears this out.

2-5 MOTION WITH CONSTANT ACCELERATION

It is fairly common to encounter motion with constant (or nearly constant) acceleration: the examples already cited of objects falling near the Earth's surface or braking cars are typical. In this section we derive a set of equations for analyzing this special case of one-dimensional kinematics with constant acceleration. Keep in mind, however, that these results can be applied *only* when the acceleration is constant and therefore do *not* apply to such situations as a swinging pendulum bob, a rocket blasting off toward Earth orbit, or a raindrop falling against air resistance. (Later in this book we will discuss methods for analyzing these situations.)

Let's assume our motion is along the x direction. We let a_x represent the x component of the acceleration vector; that is, a_x can be positive or negative. The particle's initial ($t = 0$) velocity is v_{0x} and its initial position is x_0, both of which are also the x components of vectors and can be independently positive or negative. At a later time t, the particle has velocity v_x and is located at position x. Our goal is to find the position and velocity at time t.

For constant acceleration, the instantaneous and average accelerations are everywhere equal, and so we can use Eq. 2-14 to write

$$a_x = a_{\text{av},x} = \frac{\Delta v_x}{\Delta t} = \frac{v_x - v_{0x}}{t - 0}$$

or, solving for v_x,

$$v_x = v_{0x} + a_x t. \tag{2-26}$$

This important result allows us to find the velocity at all times, but *only for constant acceleration.* Equation 2-26 gives the velocity as a function of the time, which we could write as $v_x(t)$ but which we usually write simply as v_x. Note that Eq. 2-26 is in the form of $y = mx + b$, which describes the graph of a straight line. Thus the plot of v_x against t gives a straight line of slope a_x and intercept v_{0x} (the value of v_x at $t = 0$). This line is plotted in Fig. 2-23*b*. It is apparent that Eq. 2-26 satisfies Eq. 2-25 ($a_x = dv_x/dt$).

Let us now find how the position varies with time. In this special case in which $v_x(t)$ is a straight line, the average velocity in any interval (as we have defined it in Eq. 2-22) is also equal to the average of the initial and final velocities in that interval. For the time interval from 0 to t,

$$v_{\text{av},x} = \tfrac{1}{2}(v_x + v_{0x}). \tag{2-27}$$

You can see that this must be true from the straight-line graph in Fig. 2-23*b*. Combining Eqs. 2-22, 2-26, and 2-27, we can eliminate v_x and solve for x to obtain

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2, \tag{2-28}$$

which gives us the position x at all times. We can also write Eq. 2-28 to find the displacement $x - x_0$ (the net change in position in the interval). Figure 2-23*c* shows a graph of x against t, which has the form of a parabola. Equations 2-27 and 2-28 are valid *only for constant acceleration.*

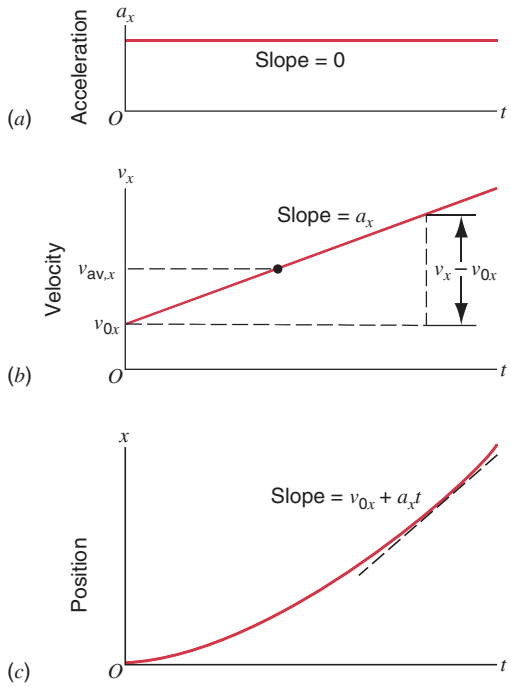

FIGURE 2-23. (*a*) The constant acceleration of a particle, equal to the (constant) slope of $v_x(t)$. (*b*) Its velocity $v_x(t)$, given at each point by the slope of the $x(t)$ curve. The average velocity $v_{\text{av},x}$, which in the case of constant acceleration is equal to the average of v_x and v_{0x}, is indicated. (*c*) The position $x(t)$ of a particle moving with constant acceleration. The curve is drawn for initial position $x_0 = 0$.

The instantaneous velocity and position should be related by $v_x = dx/dt$. Equation 2-28 satisfies that relationship, as we can show:

$$\frac{dx}{dt} = \frac{d}{dt}(x_0 + v_{0x}t + \tfrac{1}{2}a_xt^2) = v_{0x} + a_xt = v_x.$$

Students who are already familiar with integral calculus will recognize that, just as Eq. 2-26 can be obtained from Eq. 2-28 by differentiating, Eq. 2-28 can be obtained from Eq. 2-26 by integrating. We will demonstrate this process at the end of this section.

Equations 2-26 and 2-28 will serve as our basic equations for analyzing motion with constant acceleration. If we specify the *initial conditions* (the initial values of the position x_0 and the velocity v_0) and the acceleration (which later we will learn comes from the interaction of the particle with its environment), we can find the position and velocity for all values of the time t.

Keep in mind that in these equations v_x and x represent the x components of the velocity and position vectors. As is always the case in problems involving vectors, we can locate our coordinate axes anywhere we like and in any orientation. In these problems we must select the origin of the coordinate system (often chosen so that $x_0 = 0$, which simplifies the problem) and the direction of the positive x axis, so that all displacements, velocities, and accelerations in that direction are positive and those in the opposite direction are negative. Once you have chosen the origin and direction of your coordinate system, they must remain in effect throughout the solution of that problem.

Integrating the Equations of Motion (Optional)*

Equations 2-26 and 2-28, our basic expressions for kinematics with constant acceleration, can also be obtained using the methods of integral calculus. We start with the definition of acceleration, $a_x = dv_x/dt$, which we can write as

$$dv_x = a_x dt.$$

We take the integral of both sides of this equation:

$$\int dv_x = \int a_x dt = a_x \int dt,$$

where the last step, taking the acceleration out of the integral, can be made because the acceleration is constant. Carrying out the integrals, we obtain

$$v_x = a_xt + C,$$

where C is a constant of integration. We can determine the constant C from one of the initial conditions: at $t = 0$, the velocity is v_{0x}. Substituting these values into the above equation, we find $C = v_{0x}$, and so we obtain $v_x = v_{0x} + a_xt$, in agreement with Eq. 2-26.

To find $x(t)$ by integration, we begin with the definition of velocity, $v_x = dx/dt$, which we write as

$$dx = v_x dt.$$

We now substitute Eq. 2-26 for v_x and integrate on both sides:

$$\int dx = \int (v_{0x} + a_xt)\, dt = v_{0x}\int dt + a_x \int t\, dt.$$

Carrying out the integrals, we obtain

$$x = v_{0x}t + \tfrac{1}{2}a_xt^2 + C',$$

where C' is another constant of integration. To find this constant, we use the second initial condition: at $t = 0$, $x = x_0$. Substituting these values, we find $C' = x_0$ which gives $x(t)$ in agreement with Eq. 2-28. ■

*Students who are not yet familiar with integral calculus may want to delay reading this section.

SAMPLE PROBLEM 2-7. An alpha particle (the nucleus of a helium atom) travels along the inside of an evacuated straight tube 2.0 m long that forms part of a particle accelerator. The alpha particle enters the tube (at $t = 0$) moving at a velocity 9.5×10^5 m/s and emerges from the other end at time $t = 8.0 \times 10^{-7}$ s. (*a*) If the particle's acceleration is constant, what is the acceleration? (*b*) What is its velocity when it leaves the tube?

Solution (*a*) We choose the x axis to be along the tube with its positive direction to be that of the motion of the particle, and we take the origin to be at the entrance of the tube so that $x_0 = 0$. We can find the acceleration by solving Eq. 2-28 for a_x:

$$a_x = \frac{x - v_{0x}t}{\frac{1}{2}t^2}$$

$$= \frac{2.0\text{ m} - (9.5 \times 10^5\text{ m/s})(8.0 \times 10^{-7}\text{ s})}{0.5(8.0 \times 10^{-7}\text{ s})^2}$$

$$= +3.9 \times 10^{12}\text{ m/s}^2.$$

The positive sign tells us that the particle is speeding up as it passes through the tube.

(*b*) To find the velocity as the particle leaves the tube, we use Eq. 2-26:

$$v_x = v_{0x} + a_xt$$

$$= (9.5 \times 10^5\text{ m/s}) + (3.9 \times 10^{12}\text{ m/s}^2)(8.0 \times 10^{-7}\text{ s})$$

$$= +4.1 \times 10^6\text{ m/s}.$$

Consistent with the positive acceleration, the particle's velocity does increase.

SAMPLE PROBLEM 2-8. You brake your Porsche with constant acceleration from a velocity of 23.6 m/s (about 53 mph, well below the speed limit, of course) to 12.5 m/s over a distance of 105 m. (*a*) How much time elapses during this interval? (*b*) What is the acceleration? (*c*) If you were to continue braking with the same constant acceleration, how much longer would it take for you to stop and how much additional distance would you cover?

Solution (*a*) We select the positive direction for our coordinate system to be the direction of the velocity and choose the origin so that $x_0 = 0$ when you begin braking. Then the initial velocity is $v_{0x} = +23.6$ m/s at $t = 0$, and the final velocity and position are $v_x = +12.5$ m/s and $x = 105$ m at time t. Because the acceleration is constant, the average velocity in the interval can be found from the average of the initial and final velocities according to Eq. 2-27:

$$v_{\text{av},x} = \tfrac{1}{2}(v_x + v_{0x}) = \tfrac{1}{2}(12.5 \text{ m/s} + 23.6 \text{ m/s}) = 18.05 \text{ m/s}.$$

The average velocity can also be expressed as $v_{\text{av},x} = \Delta x/\Delta t$. With $\Delta x = 105$ and $\Delta t = t - 0$, we can solve for t:

$$t = \frac{\Delta x}{v_{\text{av},x}} = \frac{105 \text{ m}}{18.05 \text{ m/s}} = 5.81 \text{ s}.$$

(*b*) We can now find the acceleration from Eq. 2-26:

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{12.5 \text{ m/s} - 23.6 \text{ m/s}}{5.81 \text{ s}} = -1.91 \text{ m/s}^2.$$

The acceleration is negative, which means that, as expected, the positive velocity is growing smaller as you brake.
(*c*) Now with a known acceleration, we can find the total time for the car to go from velocity $v_{0x} = 23.6$ m/s to $v_x = 0$. Solving Eq. 2-26 for t, we find

$$t = \frac{v_x - v_{0x}}{a_x} = \frac{0 - 23.6 \text{ m/s}}{-1.91 \text{ m/s}^2} = 12.4 \text{ s}.$$

The total distance covered is found from Eq. 2-28 for this interval of time with $x_0 = 0$:

$$x = v_{0x}t + \tfrac{1}{2}a_x t^2$$
$$= (23.6 \text{ m/s})(12.4 \text{ s}) + \tfrac{1}{2}(-1.91 \text{ m/s}^2)(12.4 \text{ s})^2 = 146 \text{ m}.$$

From the time you originally began braking to the time your car came to rest, you covered a total distance of 146 m in a total time of 12.4 s. The change from 23.6 m/s to 12.5 m/s covered a distance of 105 m in 5.8 s, so the change from 12.5 m/s to 0 covered a distance of 146 m − 105 m = 41 m and lasted for a time of 12.4 s − 5.8 s = 6.6 s.

2-6 FREELY FALLING BODIES

The most common example of motion with (nearly) constant acceleration is that of a falling body near the Earth's surface. If we neglect air resistance, we find a remarkable fact: at any given point near the Earth's surface, *all bodies, regardless of their size, shape, or composition, fall with the same acceleration.* This acceleration, denoted by the symbol g, is called the *free-fall acceleration* (or sometimes the *acceleration due to gravity*). Although the acceleration depends on the distance from the center of the Earth (as we shall show in Chapter 14), if the distance of fall is small compared with the Earth's radius (6400 km) we can regard the acceleration as constant throughout the fall.

Near the Earth's surface the magnitude of g is approximately 9.8 m/s^2, a value that we use throughout the text unless we specify otherwise. The direction of the free-fall acceleration at any point establishes what we mean by the word "down" at that point.

Although we speak of *falling* bodies, bodies in upward motion experience the same free-fall acceleration (magnitude *and* direction). That is, no matter whether a particle is moving up or down, the direction of its acceleration under the influence of the Earth's gravity is always down.

The exact value of the free-fall acceleration varies with latitude and with altitude. There are also significant variations caused by differences in the local density of the Earth's crust. We discuss these variations in Chapter 14.

The equations of constant acceleration (Eqs. 2-26 and 2-28) can be applied to free fall. For this purpose, we first make two small changes: (1) We label the direction of free fall as the y axis and take its positive direction to be upward. Later, in Chapter 4, we shall consider motion in two dimensions, and we shall want to use the x label for horizontal motion. (2) We replace the constant acceleration a with $-g$, since our choice of the positive y direction to be upward means that the downward acceleration is negative. By choosing $a_y = -g$, we will always have g as a positive number.

With these small changes, the equations describing freely falling bodies are

$$v_y = v_{0y} - gt, \tag{2-29}$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2. \tag{2-30}$$

Just as we used Eqs. 2-26 and 2-28 to solve problems involving constant acceleration, we can use Eqs. 2-29 and 2-30 as the basic formulas for solving problems involving free fall.

SAMPLE PROBLEM 2-9. A body is dropped from rest and falls freely. Determine the position and velocity of the body after 1.0, 2.0, 3.0, and 4.0 s have elapsed.

Solution We choose the starting point as the origin so that $y_0 = 0$. We know the initial speed (zero) and the acceleration, and we are given the time. To find the position, we use Eq. 2-30 with $y_0 = 0$ and $v_{0y} = 0$:

$$y = -\tfrac{1}{2}gt^2.$$

Putting $t = 1.0$ s, we obtain

$$y = -\tfrac{1}{2}(9.8 \text{ m/s}^2)(1.0 \text{ s})^2 = -4.9 \text{ m}.$$

To find the velocity, we use Eq. 2-29, again with $v_{0y} = 0$:

$$v_y = -gt = -(9.8 \text{ m/s}^2)(1.0 \text{ s}) = -9.8 \text{ m/s}.$$

After falling for 1.0 s, the body is 4.9 m *below* (y is negative) its starting point and is moving *downward* (v_y is negative) with a speed of 9.8 m/s. Continuing in this way, we can find the positions and velocities at $t = 2.0$, 3.0, and 4.0 s, which are shown in Fig. 2-24.

Note that the change in velocity in each second is −9.8 m/s, and that the average velocity during each one-second interval (equal to the displacement in that interval divided by the time interval) is equal to half the sum of the initial and final velocities in the interval, as required by Eq. 2-27.

SAMPLE PROBLEM 2-10. A ball is thrown vertically upward from the ground with a speed of 25.2 m/s. (*a*) How long does it take to reach its highest point? (*b*) How high does it rise? (*c*) At what times will it be 27.0 m above the ground?

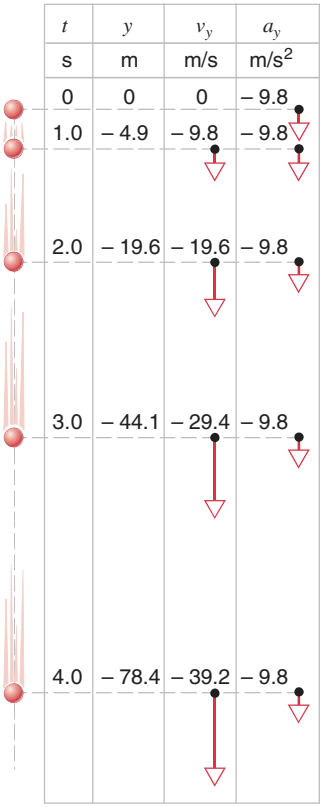

FIGURE 2-24. Sample Problem 2-9. The height, velocity, and acceleration of a body in free fall are shown.

Solution (*a*) At its highest point its velocity passes through the value zero. Given v_{0y} and $v_y (= 0)$, we wish to find t and we therefore choose Eq. 2-29, which we solve for t:

$$t = \frac{v_{0y} - v_y}{g} = \frac{25.2 \text{ m/s} - 0}{9.8 \text{ m/s}^2} = 2.57 \text{ s}.$$

(*b*) Now that we have found the time for the ball to reach its maximum height, Eq. 2-30, with y_0 assigned as 0, allows us to solve for y when we know the other quantities:

$$y = v_{0y}t - \tfrac{1}{2}gt^2$$
$$= (25.2 \text{ m/s})(2.57 \text{ s}) - \tfrac{1}{2}(9.8 \text{ m/s}^2)(2.57 \text{ s})^2 = 32.4 \text{ m}.$$

(*c*) Equation 2-30 is useful for this case, because t is the only unknown. Since we wish to solve for t, let us rewrite Eq. 2-30, with $y_0 = 0$, in the usual form of a quadratic equation:

$$\tfrac{1}{2}gt^2 - v_{0y}t + y = 0,$$

or, inserting the numerical values,

$$(4.9 \text{ m/s}^2)t^2 - (25.2 \text{ m/s})t + 27.0 \text{ m} = 0.$$

Solving this quadratic equation, we find $t = 1.52$ s and $t = 3.62$ s. At $t = 1.52$ s, the velocity of the ball is

$$v_y = v_{0y} - gt = 25.2 \text{ m/s} - (9.8 \text{ m/s}^2)(1.52 \text{ s}) = 10.3 \text{ m/s}.$$

At $t = 3.62$ s, the velocity is

$$v_y = v_{0y} - gt = 25.2 \text{ m/s} - (9.8 \text{ m/s}^2)(3.62 \text{ s}) = -10.3 \text{ m/s}.$$

The two velocities have identical magnitudes but opposite directions. You should be able to convince yourself that, in the absence of air resistance, the ball will take as long to rise to its maximum height as to fall the same distance, and that at each point it will have the same speed going up that it has coming down. Note that the answer to part (*a*) for the time to reach the highest point, 2.57 s, is exactly midway between the two times found in part (*c*). Can you explain this? Can you predict qualitatively the effect of air resistance on the times of rise and fall?

Figure 2-25 illustrates the motion of the ball. Note especially the symmetry of the upward and downward motions.

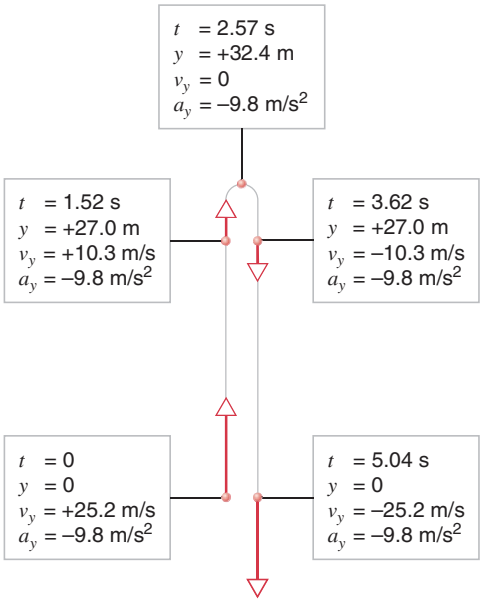

FIGURE 2-25. Sample Problem 2-10. The height, velocity, and acceleration at various points are shown.

SAMPLE PROBLEM 2-11. A rocket is launched from rest from an underwater base a distance of 125 m below the surface of a body of water. It moves vertically upward with an unknown but assumed constant acceleration (the combined effect of its engines, Earth's gravity, and the buoyancy and drag of the water), and it reaches the surface in a time of 2.15 s. When it breaks the surface its engines automatically shut off (to make it more difficult to detect) and it continues to rise. What maximum height does it reach? (Ignore any effects at the surface.)

Solution As with any projectile in free fall, we could analyze the motion of the rocket during the portion of its motion in the air if we knew the initial velocity of that part of the motion. The plan of attack in this problem is therefore to analyze the underwater portion of the motion to find the velocity when the rocket reaches the surface, and then to treat that velocity as the initial velocity of the free-fall portion. These parts must be done separately, because the acceleration changes at the surface of the water.

For the underwater motion, which is accelerated but not free fall, we can find the acceleration from Eq. 2-28 (replacing x by y) with $y - y_0 = 125$ m and $v_{0y} = 0$:

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(125 \text{ m})}{(2.15 \text{ s})^2} = 54.1 \text{ m/s}^2.$$

Now Eq. 2-26 (again with x replaced by y) gives the final velocity for this portion of the motion:

$$v_y = v_{0y} + a_y t = 0 + (54.1 \text{ m/s}^2)(2.15 \text{ s}) = 116 \text{ m/s}.$$

The velocity at the surface is 116 m/s upward. We now analyze the free-fall portion of the motion, taking this velocity to be the *initial* velocity v_{0y}. At the highest point, the rocket comes instantaneously to rest ($v_y = 0$); we use Eq. 2-29 to find the time at which this occurs:

$$t = -\frac{v_y - v_{0y}}{g} = -\frac{(0 - 116\ \text{m/s})}{9.8\ \text{m/s}^2} = 11.8\ \text{s}.$$

The height at this time is, from Eq. 2-30 with $y_0 = 0$:

$$y = v_{0y}t - \tfrac{1}{2}gt^2 = (116\ \text{m/s})(11.8\ \text{s}) - \tfrac{1}{2}(9.8\ \text{m/s}^2)(11.8\ \text{s})^2$$
$$= 687\ \text{m}.$$

To test your understanding of this problem, you should draw graphs of $y(t)$, $v_y(t)$, and $a_y(t)$ in a fashion similar to Fig. 2-17. Be sure to keep in mind which variables change continuously and smoothly and which do not in this idealized problem. How would the motion of a real rocket vary from this picture?

Measuring the Free-Fall Acceleration (Optional)

The nature of the motion of a falling object has long been of interest to scientists and philosophers. Aristotle (384–322 B.C.) thought that heavier objects would fall more rapidly because of their weight. That was the prevailing view for two millennia, until Galileo Galilei (1564–1642) made the correct assertion, that in the absence of air resistance all objects fall with the same speed. We can test this assertion by dropping a feather and a ball of lead in a vacuum, and we find that they do indeed fall at the same rate. In 1971, astronaut David Scott dropped a feather and a hammer on the (airless) Moon, and he observed that they reached the surface at about the same time.

In Galileo's time, there was no way to achieve a vacuum, and he lacked the equipment to make precise measurements of time intervals. (The famous story about Galileo dropping different objects from the Tower of Pisa and observing them to reach the ground at the same time is probably only legend. For a tower of that height, air resistance would have a considerable effect on objects unless they had exactly the same size and shape.) Galileo reduced the acceleration (making it easier to measure time intervals and also reducing the effect of air resistance) by rolling a ball down an incline instead of dropping it. He showed that in equal time intervals the ball covered distances that were proportional to the odd integers 1,3,5,7, ... The total distances covered for consecutive intervals were thus proportional to 1, 1 + 3 (= 4), 1 + 3 + 5 (= 9), 1 + 3 + 5 + 7 (= 16), and so on. He thus concluded that the distances increased as the *square* of the time, which we now know holds *only* in the case of constant acceleration. He also found that the same results were obtained regardless of the mass of the ball, and thus (in present-day terminology) he deduced that the free-fall acceleration is independent of the mass of the object.

Today the measurement of g is a standard exercise in the introductory laboratory. By timing the fall of an object over a distance of a meter or two (which takes about 0.5 s), it is possible to determine g to a precision of a few percent. Using even a crude pendulum (which "slows down" the motion in analogy with Galileo's incline) and measuring the time for one complete back-and-forth swing, you can (as we discuss in Chapter 17) determine g to a precision of about 0.1%. This level of precision is sufficient to observe the variation in g between sea level and a high mountain (3 km or 10,000 ft), or between the equator and the poles of the Earth.

With carefully designed apparatus, the pendulum method can be extended to a precision of 1 part in 10^6, sufficient to detect variations in g from one floor of a building to the next. To achieve even greater precision, investigators have refined the free-fall method. By dropping an object in vacuum and reflecting a laser beam from it as it falls, very precise determinations of the distance of fall can be made. Coupled with atomic clocks to measure the time of fall, the value of g can be determined to a precision of about 1 part in 10^9, which enables the variation in g over a vertical distance of 1 cm to be observed. Equivalently, such a gravity meter can detect the gravitational effect of the measuring scientist standing 1 m from the apparatus!

Falling-body gravimeters for making these precise measurements are now commercially available. In the latest model, the falling object is placed in an evacuated box and the object is projected upward, so that measurements can be taken as the object rises and as it falls, as suggested in Problem 33. Figure 2-26 shows a portable version of this type of apparatus.

FIGURE 2-26. A portable rise-and-fall gravimeter. Its uses include geophysical research, oil and mineral exploration, and inertial navigation. Photo provided courtesy of Dr. T.M. Niebauer, Micro-g Solutions. (See http://www.microgsolutions.com.)

Such accurate measurements of the free-fall acceleration permit detailed studies of the Earth's gravity, which has important practical consequences. Variations in g from place to place can reveal the presence of oil or minerals under the Earth's surface, and variations in g with time can reveal movement of the Earth's plates or seismic activity. Knowledge of the small variations in g due to irregularities in the Earth's gravity enables accurate calculations of the paths of ballistic missiles or Earth satellites. In addition to these practical applications, precise measurements of g can provide detailed tests of our understanding of the theory of gravitation, one of the basic forces of the universe. ■

MULTIPLE CHOICE

2-1 Kinematics with Vectors

2-2 Properties of Vectors

2-3 Position, Velocity, and Acceleration Vectors

1. An object is moving with velocity given by $\vec{\mathbf{v}}(t) = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}} + v_z(t)\hat{\mathbf{k}}$ where $v_z(t) = 0$. From this, one can conclude

(a) that the acceleration $\vec{\mathbf{a}}(t)$

(A) will have no components that are identically zero.
(B) may have some components that are identically zero.
(C) will have only a z component that is identically zero.
(D) will have an identically zero z component, and maybe an identically zero component in the x or y direction.

(b) and that the position $\vec{\mathbf{r}}(t)$

(A) will have no components that are identically zero.
(B) may have some components that are identically zero.
(C) will have only a z component that is identically zero.
(D) will have an identically zero z component, and maybe an identically zero component in the x or y direction.

2. An object is moving in the xy plane with the position as a function of time given by $\vec{\mathbf{r}} = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$. Point O is at $\vec{\mathbf{r}} = 0$. The object is definitely moving toward O when

(A) $v_x > 0, v_y > 0$. (B) $v_x < 0, v_y < 0$.
(C) $xv_x + yv_y < 0$. (D) $xv_x + yv_y > 0$.

2-4 One-Dimensional Kinematics

3. An object is launched straight up into the air from the ground with an initial vertical velocity of 30 m/s. The object rises to a highest point approximately 45 m above the ground in 3 seconds; it then falls back to the ground in 3 more seconds, impacting with a speed of 30 m/s.

(a) The average *speed* of the object during the 6-second interval is closest to

(A) 0 m/s. (B) 5 m/s. (C) 15 m/s. (D) 30 m/s.

(b) The magnitude of the average *velocity* during the 6-second interval is closest to

(A) 0 m/s. (B) 5 m/s. (C) 15 m/s. (D) 30 m/s.

4. A car travels 15 miles east at a constant speed of 20 mi/h, then continues east for 20 miles at a constant 30 mi/h. What can be concluded about the magnitude of the average velocity?

(A) $v_{av} < 25$ mi/h. (B) $v_{av} = 25$ mi/h.
(C) $v_{av} > 25$ mi/h.
(D) More information is needed to answer the question.

5. An object is moving along the x axis with position as a function of time given by $x = x(t)$. Point O is at $x = 0$. The object is definitely moving toward O when

(A) $dx/dt < 0$. (B) $dx/dt > 0$.
(C) $d(x^2)/dt < 0$. (D) $d(x^2)/dt > 0$.

6. An object starts from rest at $x = 0$ when $t = 0$. The object moves in the x direction with positive velocity after $t = 0$. The instantaneous velocity and average velocity are related by

(A) $dx/dt < x/t$. (B) $dx/dt = x/t$. (C) $dx/dt > x/t$.
(D) dx/dt can be larger than, smaller than, or equal to x/t.

7. Figure 2-27 shows several graphs with unlabeled axes. (a) Which graph would best represent velocity as a function of time for an object moving with constant speed? (b) Which graph best represents velocity as a function of time for acceleration given by $a = +3t$? (c) Which graph best represents distance as a function of time for a constant negative acceleration? (d) Which graph best represents velocity as a function of time if graph E shows distance as a function of time?

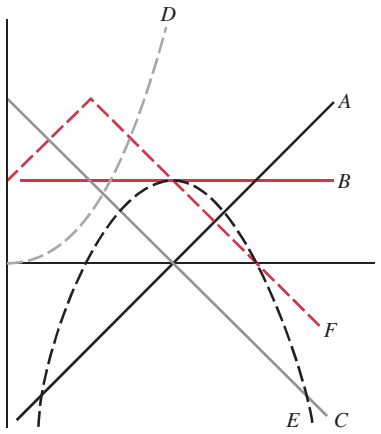

FIGURE 2-27. Multiple-choice question 7.

2-5 Motion with Constant Acceleration

8. An object is moving in the x direction with velocity $v_x(t)$, and dv_x/dt is a nonzero constant. With $v_x = 0$ when $t = 0$, then for $t > 0$ the quantity $v_x\, dv_x/dt$ is

(A) negative. (B) zero. (C) positive.
(D) not determined from the information given.

2-6 Freely Falling Bodies

9. A fellow student makes the following comment during a study session: "A freely falling object falls a greater distance during each second than the total distance fallen in all the previous seconds." This statement

(A) is always true.
(B) is true only for sufficiently short times.
(C) is true for sufficiently long times.
(D) is never true.

10. An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention *up is positive,* how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object?

(A) On the way up $a_y > 0$, on the way down $a_y > 0$.
(B) On the way up $a_y < 0$, on the way down $a_y > 0$.
(C) On the way up $a_y > 0$, on the way down $a_y < 0$.
(D) On the way up $a_y < 0$, on the way down $a_y < 0$.

11. A boy jumps off a high diving board above a swimming pool. Halfway between the board and the water he tosses a ball upward. Ignoring air friction, the instant after the ball leaves his hands the vertical component of the acceleration of the ball
(A) is positive, but then decreases through zero to -9.8 m/s^2.
(B) is zero, but then decreases to -9.8 m/s^2.
(C) is between zero and -9.8 m/s^2, but then decreases to -9.8 m/s^2.
(D) is -9.8 m/s^2.

12. A small toy looks like a pipe and shoots a marble out each end. The toy is dropped out of a tree and fires halfway to the ground. One marble shoots directly up, the other directly down. Consider the vertical component of the acceleration a_y of the marbles immediately after the marbles leave the toy, ignoring any air friction.
(A) The upward-moving marble has $a_y < -9.8$ m/s^2.
(B) The upward-moving marble has $a_y = -9.8$ m/s^2.
(C) The upward-moving marble has $a_y > -9.8$ m/s^2.
(D) The downward-moving marble has $a_y > -9.8$ m/s^2.

QUESTIONS

1. Can two vectors having different magnitudes be combined to give a zero resultant? Can three vectors?

2. Can a vector have zero magnitude if one of its components is not zero?

3. Can the sum of the magnitudes of two vectors ever be equal to the magnitude of the sum of these two vectors?

4. Can the magnitude of the difference between two vectors ever be greater than the magnitude of either vector? Can it be greater than the magnitude of their sum? Give examples.

5. Suppose that $\vec{\mathbf{d}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2$. Does this mean that we must have either $d \geq d_1$ or $d \geq d_2$? If not, explain why.

6. Can the speed of a particle ever be negative? If so, give an example; if not, explain why.

7. Does average velocity have a direction associated with it?

8. Each second a rabbit moves one-half the remaining distance from its nose to a head of lettuce. Does the rabbit ever get to the lettuce? What is the limiting value of the rabbit's average velocity? Draw graphs showing the rabbit's velocity and position as time increases.

9. Instead of the definition given in Eq. 2-13, we might have defined average speed as the magnitude of the average velocity. Are the definitions different? Give examples to support your answer.

10. A racing car, in a qualifying two-lap heat, covers the first lap with an average speed of 90 mi/h. The driver wants to speed up during the second lap so that the average speed of the two laps together will be 180 mi/h. Show that it cannot be done.

11. Bob beats Judy by 10 m in a 100-m dash. Bob, claiming to give Judy an equal chance, agrees to race her again but to start from 10 m behind the starting line. Does this really give Judy an equal chance?

12. When the velocity is constant, can the average velocity over any time interval differ from the instantaneous velocity at any instant? If so, give an example; if not, explain why.

13. Can the average velocity of a particle moving along the x axis ever be $\frac{1}{2}(v_{0x} + v_x)$ if the acceleration is not constant? Prove your answer with the use of graphs.

14. (*a*) Can an object have zero velocity and still be accelerating? (*b*) Can an object have a constant velocity and still have a varying speed? In each case, give an example if your answer is yes; explain why if your answer is no.

15. Can the velocity of an object reverse direction when its acceleration is constant? If so, give an example; if not, explain why.

16. Figure 2-28 shows Colonel John P. Stapp in his braking rocket sled; see Exercise 45. (*a*) His body is an accelerometer, not a speedometer. Explain. (*b*) Can you tell the direction of the acceleration from the figure?

FIGURE 2-28. Question 16 and Exercise 45.

17. Can an object be increasing in speed as the magnitude of its acceleration decreases? If so, give an example; if not, explain why.

18. Of the following situations, which one is impossible? (*a*) A body having velocity east and acceleration east; (*b*) a body having velocity east and acceleration west; (*c*) a body having zero velocity but acceleration not zero; (*d*) a body having constant acceleration and variable velocity; (*e*) a body having constant velocity and variable acceleration.

19. If a particle is released from rest ($v_{0x} = 0$) at $x_0 = 0$ at the time $t = 0$, Eq. 2-28 for constant acceleration says that it is at position x at two different times—namely, $+\sqrt{2x/a_x}$ and $-\sqrt{2x/a_x}$. What is the meaning of the negative root of this quadratic equation?

20. What happens to our kinematic equations (Eq. 2-26 or 2-28) under the operation of time reversal—that is, replacing t by $-t$? Explain.

21. We expect a truly general relation, such as Eqs. 2-26 and 2-28, to be valid regardless of the choice of coordinate system. By

demanding that general equations be dimensionally consistent we ensure that the equations are valid regardless of the choice of units. Is there any need then for units or coordinate systems?

22. What are some examples of falling objects for which it would be unreasonable to neglect air resistance?

23. Figure 2-29 shows a shot tower in Baltimore, Maryland. It was built in 1829 and used to manufacture lead shot pellets by pouring molten lead through a sieve at the top of the tower. The lead pellets solidify as they fall into a tank of water at the bottom of the tower, 230 ft below. What are the advantages of manufacturing shot in this way?

FIGURE 2-29. Question 23.

24. A person standing on the edge of a cliff at some height above the ground throws one ball straight up with initial speed v_0 and then throws another ball straight down with the same initial speed. Which ball, if either, has the larger speed when it hits the ground? Neglect air resistance.

25. What is the downward acceleration of a projectile that is released from a missile accelerating upward at 9.8 m/s^2?

26. On another planet, the value of g is one-half the value on Earth. How is the time needed for an object to fall to the ground from rest on that planet related to the time required to fall the same distance on Earth?

27. (*a*) A stone is thrown upward with a certain speed on a planet where the free-fall acceleration is double that on Earth. How high does it rise compared to the height it rises on Earth? (*b*) If the initial speed were doubled, what change would that make?

28. Consider a ball thrown vertically up. Taking air resistance into account, would you expect the time during which the ball rises to be longer or shorter than the time during which it falls? Why?

29. Make a qualitative graph of speed v versus time t for a falling object (*a*) for which air resistance can be ignored and (*b*) for which air resistance cannot be ignored.

30. A second ball is dropped down an elevator shaft 1 s after the first ball is dropped. (*a*) What happens to the distance between the balls as time goes on? (*b*) How does the ratio v_1/v_2 of the speed of the first ball to the speed of the second ball change as time goes on? Neglect air resistance, and give qualitative answers.

31. Repeat Question 30 taking air resistance into account. Again, give qualitative answers.

32. If m is a light stone and M is a heavy one, according to Aristotle M should fall faster than m. Galileo attempted to show that Aristotle's belief was logically inconsistent by the following argument. Tie m and M together to form a double stone. Then, in falling, m should retard M, because it tends to fall more slowly, and the combination would fall faster than m but more slowly than M; but according to Aristotle the double body $(M + m)$ is heavier than M and, hence, should fall faster than M. If you accept Galileo's reasoning as correct, can you conclude that M and m must fall at the same rate? What need is there for experiment in that case? If you believe Galileo's reasoning is incorrect, explain why.

EXERCISES

2-1 Kinematics with Vectors

2-2 Properties of Vectors

1. Consider two displacements, one of magnitude 3 m and another of magnitude 4 m. Show how the displacement vectors may be combined to get a resultant displacement of magnitude (*a*) 7 m, (*b*) 1 m, and (*c*) 5 m.

2. A person walks in the following pattern: 3.1 km north, then 2.4 km west, and finally 5.2 km south. (*a*) Construct the vector diagram that represents this motion. (*b*) How far and in what direction would a bird fly in a straight line to arrive at the same final point?

3. Vector $\vec{\mathbf{a}}$ has a magnitude of 5.2 units and is directed east. Vector $\vec{\mathbf{b}}$ has a magnitude of 4.3 units and is directed 35° west of north. By constructing vector diagrams, find the magnitudes and directions of (a) $\vec{\mathbf{a}} + \vec{\mathbf{b}}$, and (b) $\vec{\mathbf{a}} - \vec{\mathbf{b}}$.

4. (*a*) What are the components of a vector $\vec{\mathbf{a}}$ in the xy plane if its direction is 252° counterclockwise from the positive x axis and its magnitude is 7.34 units? (*b*) The x component of a certain vector is -25 units and the y component is $+43$ units. What are the magnitude of the vector and the angle between its direction and the positive x axis?

5. A person desires to reach a point that is 3.42 km from her present location and in a direction that is 35.0° north of east.

However, she must travel along streets that go either north-south or east-west. What is the minimum distance she could travel to reach her destination?

6. A ship sets out to sail to a point 124 km due north. An unexpected storm blows the ship to a point 72.6 km to the north and 31.4 km to the east of its starting point. How far, and in what direction, must it now sail to reach its original destination?

7. (*a*) What is the sum in unit-vector notation of the two vectors $\vec{\mathbf{a}} = 5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\vec{\mathbf{b}} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$? (*b*) What are the magnitude and the direction of $\vec{\mathbf{a}} + \vec{\mathbf{b}}$?

8. Two vectors are given by $\vec{\mathbf{a}} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. Find (*a*) $\vec{\mathbf{a}} + \vec{\mathbf{b}}$, (*b*) $\vec{\mathbf{a}} - \vec{\mathbf{b}}$, and (*c*) a vector $\vec{\mathbf{c}}$ such that $\vec{\mathbf{a}} - \vec{\mathbf{b}} + \vec{\mathbf{c}} = 0$.

9. Given two vectors, $\vec{\mathbf{a}} = 4.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}}$ and $\vec{\mathbf{b}} = 6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}$, find the magnitudes and directions (with the $+x$ axis) of (*a*) $\vec{\mathbf{a}}$, (*b*) $\vec{\mathbf{b}}$, (*c*) $\vec{\mathbf{a}} + \vec{\mathbf{b}}$, (*d*) $\vec{\mathbf{b}} - \vec{\mathbf{a}}$, and (*e*) $\vec{\mathbf{a}} - \vec{\mathbf{b}}$.

10. Two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ have equal magnitudes of 12.7 units. They are oriented as shown in Fig. 2-30 and their vector sum is $\vec{\mathbf{r}}$. Find (*a*) the x and y components of $\vec{\mathbf{r}}$, (*b*) the magnitude of $\vec{\mathbf{r}}$, and (*c*) the angle $\vec{\mathbf{r}}$ makes with the $+x$ axis.

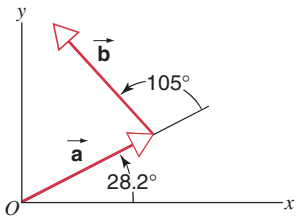

FIGURE 2-30. Exercise 10.

2-3 Position, Velocity, and Acceleration Vectors

11. A woman walks 250 m in the direction 35° east of north, then 170 m directly east. (*a*) Using graphical methods, find her final displacement from the starting point. (*b*) Compare the magnitude of her displacement with the distance she walked.

12. A car is driven east for a distance of 54 km, then north for 32 km, and then in a direction 28° east of north for 27 km. Draw the vector diagram and determine the total displacement of the car from its starting point.

13. The minute hand of a wall clock measures 11.3 cm from axis to tip. What is the displacement vector of its tip (*a*) from a quarter after the hour to half past, (*b*) in the next half hour, and (*c*) in the next hour?

14. A particle undergoes three successive displacements in a plane, as follows: 4.13 m southwest, 5.26 m east, and 5.94 m in a direction 64.0° north of east. Choose the x axis pointing east and the y axis pointing north and find (*a*) the components of each displacement, (*b*) the components of the resultant displacement, (*c*) the magnitude and direction of the resultant displacement, and (*d*) the displacement that would be required to bring the particle back to the starting point.

15. A radar station detects a missile approaching from the east. At first contact, the range to the missile is 12,000 ft at 40.0° above the horizon. The missile is tracked for another 123° in the east-west plane, the range at final contact being 25,800 ft; see Fig. 2-31. Find the displacement of the missile during the period of radar contact.

16. A plane flies 410 mi east from city *A* to city *B* in 45 min and then 820 mi south from city *B* to city *C* in 1 h 30 min. (*a*) What are the magnitude and direction of the displacement vector that represents the total trip? What are (*b*) the average velocity vector and (*c*) the average speed for the trip?

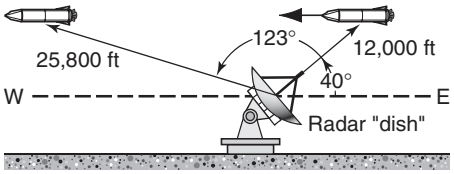

FIGURE 2-31. Exercise 15.

17. The position of a particle moving in an xy plane is given by $\vec{\mathbf{r}} = [(2\text{ m/s}^3)t^3 - (5\text{ m/s})t]\hat{\mathbf{i}} + [(6\text{ m}) - (7\text{ m/s}^4)t^4]\hat{\mathbf{j}}$. Calculate (*a*) $\vec{\mathbf{r}}$, (*b*) $\vec{\mathbf{v}}$, and (*c*) $\vec{\mathbf{a}}$ when $t = 2$ s.

18. In 3 h 24 min, a balloon drifts 8.7 km north, 9.7 km east, and 2.9 km in elevation from its release point on the ground. Find (*a*) the magnitude of its average velocity and (*b*) the angle its average velocity makes with the horizontal.

19. The velocity of a particle moving in the xy plane is given by $\vec{\mathbf{v}} = [(6.0\text{ m/s}^2)t - (4.0\text{ m/s}^3)t^2]\hat{\mathbf{i}} + (8.0\text{ m/s})\hat{\mathbf{j}}$. Assume $t > 0$. (*a*) What is the acceleration when $t = 3$ s? (*b*) When (if ever) is the acceleration zero? (*c*) When (if ever) is the velocity zero? (*d*) When (if ever) does the speed equal 10 m/s?

20. A particle is moving in the xy plane with velocity $\vec{\mathbf{v}}(t) = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}$ and acceleration $\vec{\mathbf{a}}(t) = a_x(t)\hat{\mathbf{i}} + a_y(t)\hat{\mathbf{j}}$. By taking the appropriate derivative, show that the magnitude of $\vec{\mathbf{v}}$ can be constant only if $a_x v_x + a_y v_y = 0$.

2-4 One-Dimensional Kinematics

21. A plane flies round-trip between Los Angeles and Namulevu, Vanuavinaka. The plane takes off at 12:50 P.M. Los Angeles time and lands at 6:50 P.M. Namulevu time. On the return trip it takes off at 1:50 A.M. Namulevu time and lands at 6:50 P.M. Los Angeles time. Assume that the flight time is the same in both directions and that the plane flies in a straight line at an average speed of 520 mi/hr. (*a*) What length of time is the flight (one way, as measured by the passengers)? (*b*) What is the time difference between Namulevu and Los Angeles? (*c*) Approximately where on the globe is Namulevu located?

22. On April 15 an airplane takes off at 4:40 P.M. from Belém, Brazil bound for Villamil, Ecuador (in the Galapagos). The plane lands at 8:40 P.M. Villamil local time. The sun sets at 6:15 P.M. in Belém (local time), and 7:06 P.M. in Villamil (local time). At what time during the flight do the airplane passengers see the sun set?

23. How far does your car, moving at 70 mi/h (= 112 km/h) travel forward during the 1 s of time that you take to look at an accident on the side of the road?

24. New York Yankees pitcher Roger Clemens threw a fastball at a horizontal speed of 160 km/h, as verified by a radar gun. How long did it take for the ball to reach home plate, which is 18.4 m away?

25. Figure 2-32 shows the relation between the age of the oldest sediment, in millions of years, and the distance, in kilometers, at which the sediment was found from a particular ocean ridge. Seafloor material is extruded from this ridge and moves away from it at approximately uniform speed. Find the speed, in centimeters per year, at which this material recedes from the ridge.

26. Maurice Greene once ran the 100-m dash in 9.81 s (the wind was at his back), and Khalid Khannouchi ran the marathon (26 mi, 385 yd) in 2:05:42. (*a*) What are their average speeds?

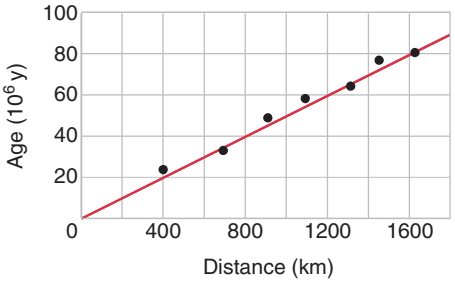

FIGURE 2-32. Exercise 25.

(*b*) If Maurice Greene could maintain his sprint speed during a marathon, how long would it take him to finish?

27. The legal speed limit on a highway is changed from 55 mi/h (= 88.5 km/h) to 65 mi/h (= 104.6 km/h). How much time is thereby saved on a trip from the Buffalo entrance to the New York City exit of the New York State Thruway for someone traveling at the higher speed over this 435-mi (= 700-km) stretch of highway?

28. A high-performance jet plane, practicing radar avoidance maneuvers, is in horizontal flight 35 m above the level ground. Suddenly, the plane encounters terrain that slopes gently upward at 4.3°, an amount difficult to detect; see Fig. 2-33. How much time does the pilot have to make a correction if the plane is to avoid flying into the ground? The airspeed is 1300 km/h.

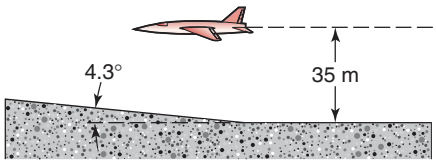

FIGURE 2-33. Exercise 28.

29. A car travels up a hill at the constant speed of 40 km/h and returns down the hill at the speed of 60 km/h. Calculate the average speed for the round trip.

30. Compute your average speed in the following two cases. (*a*) You walk 240 ft at a speed of 4.0 ft/s and then run 240 ft at a speed of 10 ft/s along a straight track. (*b*) You walk for 1.0 min at a speed of 4.0 ft/s and then run for 1.0 min at 10 ft/s along a straight track.

31. How far does the runner whose velocity-time graph is shown in Fig. 2-34 travel in 16 s?

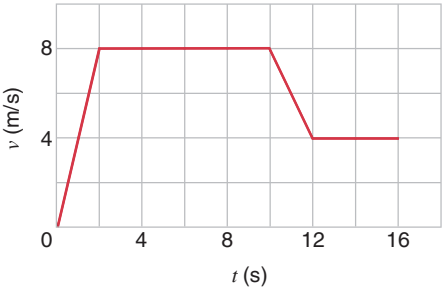

FIGURE 2-34. Exercises 31 and 32.

32. What is the acceleration of the runner in Exercise 31 at $t = 11$ s?

33. A particle had a velocity of 18 m/s in the $+x$ direction and 2.4 s later its velocity was 30 m/s in the opposite direction. What was the average acceleration of the particle during this 2.4-s interval?

34. An object moves in a straight line as described by the velocity-time graph in Fig. 2-35. Sketch a graph that represents the acceleration of the object as a function of time.

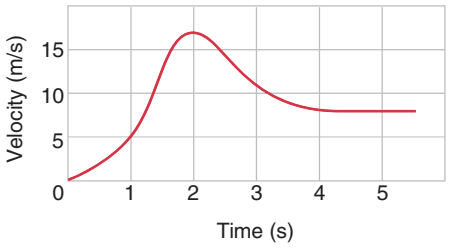

FIGURE 2-35. Exercise 34.

35. The graph of x versus t in Fig. 2-36*a* is for a particle in straight line motion. (*a*) State for each interval whether the velocity v_x is +, −, or 0, and whether the acceleration a_x is +, −, or 0. The intervals are *OA*, *AB*, *BC*, and *CD*. (*b*) From the curve, is there any interval over which the acceleration is obviously not constant? (Ignore the behavior at the endpoints of the intervals.)

36. Answer the previous questions for the motion described by the graph of Fig. 2-36*b*.

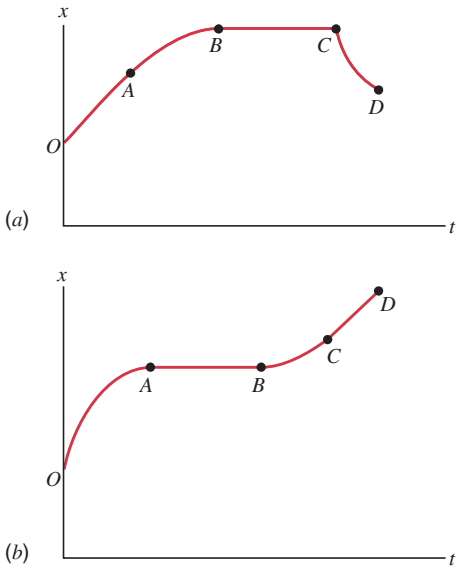

FIGURE 2-36. (*a*) Exercise 35 and (*b*) Exercise 36.

37. A particle moves along the x axis with a displacement versus time as shown in Fig. 2-37. Roughly sketch curves of velocity versus time and acceleration versus time for this motion.

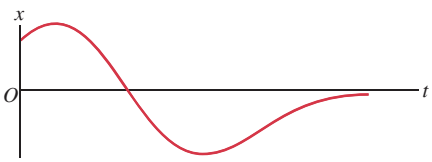

FIGURE 2-37. Exercise 37.

38. A particle moving along the positive x axis has the following positions at various times:

x(m)	0.080	0.050	0.040	0.050	0.080	0.13	0.20
t(s)	0	1	2	3	4	5	6

(a) Plot displacement (not position) versus time. (b) Find the average velocity of the particle in the intervals 0 to 1 s, 0 to 2 s, 0 to 3 s, 0 to 4 s. (c) Find the slope of the curve drawn in part (a) at the points $t = 0$, 1, 2, 3, 4, and 5 s. (d) Plot the slope (units?) versus time. (e) From the curve of part (d) determine the acceleration of the particle at times $t = 2$, 3, and 4 s.

39. The position of a particle along the x axis depends on the time according to the equation $x = At^2 - Bt^3$, where x is in meters and t is in seconds. (a) What SI units must A and B have? For the following, let their numerical values in SI units be 3.0 and 1.0, respectively. (b) At what time does the particle reach its maximum positive x position? (c) What total path-length does the particle cover in the first 4 seconds? (d) What is its displacement during the first 4 seconds? (e) What is the particle's velocity at the end of each of the first 4 seconds? (f) What is the particle's acceleration at the end of each of the first 4 seconds? (g) What is the average velocity for the time interval $t = 2$ to $t = 4$ s?

2-5 Motion with Constant Acceleration

40. A jumbo jet needs to reach a speed of 360 km/h (= 224 mi/h) on the runway for takeoff. Assuming a constant acceleration and a runway 1.8 km (= 1.1 mi) long, what minimum acceleration from rest is required?

41. A rocket ship in free space moves with constant acceleration equal to 9.8 m/s^2. (a) If it starts from rest, how long will it take to acquire a speed one-tenth that of light? (b) How far will it travel in so doing? (The speed of light is 3.0×10^8 m/s.)

42. The head of a rattlesnake can accelerate 50 m/s^2 in striking a victim. If a car could do as well, how long would it take for it to reach a speed of 100 km/h from rest?

43. A muon (an elementary particle) is shot with initial speed 5.20×10^6 m/s into a region where an electric field produces an acceleration of 1.30×10^{14} m/s^2 directed opposite to the initial velocity. How far does the muon travel before coming to rest?

44. An electron with initial velocity $v_0 = 1.5 \times 10^5$ m/s enters a region 1.2 cm long where it is electrically accelerated (see Fig. 2-38). It emerges with a velocity $v = 5.8 \times 10^6$ m/s. What was its acceleration, assumed constant? (Such a process occurs in the electron gun in a cathode-ray tube, used in television receivers and computer monitors.)

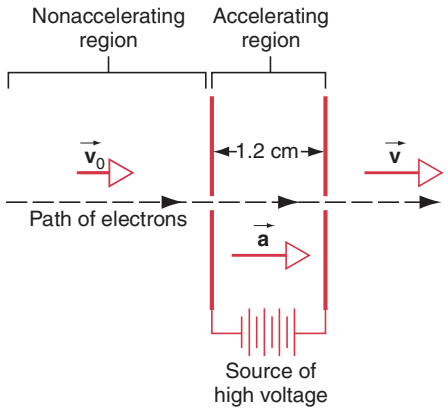

FIGURE 2-38. Exercise 44.

45. A world's land speed record was set by Colonel John P. Stapp when, on March 19, 1954, he rode a rocket-propelled sled that moved down a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s; see Fig. 2-28. What acceleration did he experience? Express your answer in terms of g (= 9.8 m/s^2), the acceleration due to gravity. (Note that his body acts as an accelerometer, not a speedometer.)

46. The brakes on your automobile are capable of creating a deceleration of 17 ft/s^2. If you are going 85 mi/h and suddenly see a state trooper, what is the minimum time in which you can get your car under the 55 mi/h speed limit?

47. On a dry road a car with good tires may be able to brake with a deceleration of 11.0 mi/h/s (= 4.92 m/s^2). (a) How long does it take such a car, initially traveling at 55 mi/h (= 24.6 m/s), to come to rest? (b) How far does it travel in this time?

48. An arrow is shot straight up into the air and on its return strikes the ground at 260 ft/s, imbedding itself 9.0 in. into the ground. Find (a) the acceleration (assumed constant) required to stop the arrow, and (b) the time required for the ground to bring it to rest.

49. An elevator cab in the New York Marriott Marquis (see Fig. 2-39) has a total run of 624 ft. Its maximum speed is 1000 ft/min and its (constant) acceleration is 4.00 ft/s^2. (a) How far does it move while accelerating to full speed from rest? (b) How long does it take to make the run, starting and ending at rest?

FIGURE 2-39. Exercise 49.

50. An automobile traveling 35 mi/h (= 56 km/h) is 110 ft (= 34 m) from a barrier when the driver slams on the brakes. Four seconds later the car hits the barrier. (a) What was the automobile's constant deceleration before impact? (b) How fast was the car traveling at impact?

2-6 Freely Falling Bodies

51. Raindrops fall to the ground from a cloud 1700 m above Earth's surface. If they were not slowed by air resistance, how

fast would the drops be moving when they struck the ground? Would it be safe to walk outside during a rainstorm?

52. The single cable supporting an unoccupied construction elevator breaks when the elevator is at rest at the top of a 120-m-high building. (*a*) With what speed does the elevator strike the ground? (*b*) For how long was it falling? (*c*) What was its speed when it passed the halfway point on the way down? (*d*) For how long was it falling when it passed the halfway point?

53. At a construction site a pipe wrench strikes the ground with a speed of 24.0 m/s. (*a*) From what height was it inadvertently dropped? (*b*) For how long was it falling?

54. (*a*) With what speed must a ball be thrown vertically up in order to rise to a maximum height of 53.7 m? (*b*) For how long will it be in the air?

55. A rock is dropped from a 100-m-high cliff. How long does it take to fall (*a*) the first 50.0 m and (*b*) the second 50.0 m?

56. Space explorers land on a planet in our solar system. They note that a small rock tossed at 14.6 m/s vertically upward takes 7.72 s to return to the ground. On which planet have they landed? (Hint: See Appendix C.)

57. A ball thrown straight up takes 2.25 s to reach a height of 36.8 m. (*a*) What was its initial speed? (*b*) What is its speed at this height? (*c*) How much higher will the ball go?

58. A ball is dropped from a height of 2.2 m and rebounds to a height of 1.9 m above the floor. Assume the ball was in contact with the floor for 96 ms and determine the average acceleration (magnitude and direction) of the ball during contact with the floor.

59. Two objects begin a free fall from rest from the same height 1.00 s apart. How long after the first object begins to fall will the two objects be 10.0 m apart?

60. A balloon is ascending at 12.4 m/s at a height of 81.3 m above the ground when a package is dropped. (*a*) With what speed does the package hit the ground? (*b*) How long did it take to reach the ground?

61. A dog sees a flowerpot sail up and then back down past a window 1.1 m high. If the total time the pot is in sight is 0.54 s, find the height above the top of the window to which the pot rises.

PROBLEMS

1. Rock *faults* are ruptures along which opposite faces of rock have moved past each other, parallel to the fracture surface. Earthquakes often accompany this movement. In Fig. 2-40, points *A* and *B* coincided before faulting. The component of the net displacement *AB* parallel to the horizontal surface fault line is called the *strike-slip* (*AC*). The component of the net displacement along the steepest line of the fault plane is the *dip-slip* (*AD*). (*a*) What is the net shift if the strike-slip is 22 m and the dip-slip is 17 m? (*b*) If the fault plane is inclined 52° to the horizontal, what is the net *vertical* displacement of *B* as a result of the faulting in (*a*)?

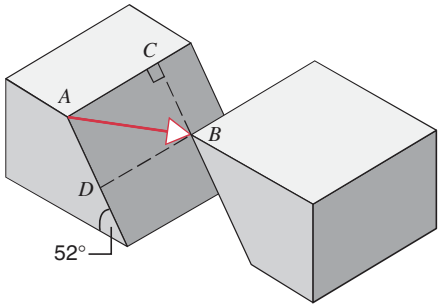

FIGURE 2-40. Problem 1.

2. A wheel with a radius of 45 cm rolls without slipping along a horizontal floor, as shown in Fig. 2-41. *P* is a dot painted on the rim of the wheel. At time t_1, *P* is at the point of contact between the wheel and the floor. At a later time t_2, the wheel has rolled through one-half of a revolution. What is the displacement of *P* during this interval?

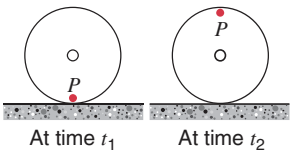

FIGURE 2-41. Problem 2.

3. A room has the dimensions 10 ft × 12 ft × 14 ft. A fly starting at one corner ends up at a diametrically opposite corner. (*a*) Find the displacement vector in a frame with coordinate axes parallel to the edges of the room. (*b*) What is the magnitude of the displacement? (*c*) Could the length of the path traveled by the fly be less than this distance? Greater than this distance? Equal to this distance? (*d*) If the fly walks rather than flies, what is the length of the shortest path it can take?

4. Two vectors of magnitudes *a* and *b* make an angle θ with each other when placed tail to tail. Prove, by taking components along two perpendicular axes, that the magnitude of their sum is

$$r = \sqrt{a^2 + b^2 + 2ab\cos\theta}.$$

5. You drive on Interstate 10 from San Antonio to Houston, one-half the time at 35.0 mi/h (= 56.3 km/h) and the other half at 55.0 mi/h (= 88.5 km/h). On the way back you travel one-half the distance at 35.0 mi/h and the other half at 55.0 mi/h. What is your average speed (*a*) from San Antonio to Houston, (*b*) from Houston back to San Antonio, and (*c*) for the entire trip?

6. The position of an object moving in a straight line is given by $x = At + Bt^2 + Ct^3$, where $A = 3.0$ m/s, $B = -4.0$ m/s^2, and $C = 1.0$ m/s^3. (*a*) What is the position of the object at $t = 0$, 1, 2, 3, and 4 s? (*b*) What is the object's displacement between $t = 0$ and $t = 2$ s? Between $t = 0$ and $t = 4$ s? (*c*) What is the average velocity for the time interval from $t = 2$ to $t = 4$ s? From $t = 0$ to $t = 3$ s?

7. Two trains, each having a speed of 34 km/h, are headed toward each other on the same straight track. A bird that can fly 58

km/h flies off the front of one train when they are 102 km apart and heads directly for the other train. On reaching the other train it flies directly back to the first train, and so forth. (*a*) How many trips can the bird make from one train to the other before the trains crash? (*b*) What is the total distance the bird travels?

8. An object, constrained to move along the x axis, travels a distance d_1 with constant velocity v_1 for a time t_1. It then instantaneously changes its velocity to a constant v_2 for a time t_2, traveling a distance d_2. (*a*) Show that

$$\frac{v_1 d_1 + v_2 d_2}{d_1 + d_2} \geq \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}.$$

(*b*) Under what conditions is this an equality?

9. The position of a particle moving along the x axis is given by $x = A + Bt^3$, where $A = 9.75$ cm and $B = 1.50$ cm/s^3. Consider the time interval $t = 2$ to $t = 3$ s and calculate (*a*) the average velocity; (*b*) the instantaneous velocity at $t = 2$ s; (*c*) the instantaneous velocity at $t = 3$ s; (*d*) the instantaneous velocity at $t = 2.5$ s; and (*e*) the instantaneous velocity when the particle is midway between its positions at $t = 2$ and $t = 3$ s.

10. For each of the following situations, sketch a graph that is a possible description of position as a function of time for a particle that moves along the x axis. At $t = 1$ s, the particle has (*a*) zero velocity and positive acceleration; (*b*) zero velocity and negative acceleration; (*c*) negative velocity and positive acceleration; (*d*) negative velocity and negative acceleration. (*e*) For which of these situations is the speed of the particle increasing at $t = 1$ s?

11. If the position of an object is given by $x = (2.0 \text{ m/s}^3)t^3$, find (*a*) the average velocity and the average acceleration between $t = 1$ and $t = 2$ s and (*b*) the instantaneous velocities and the instantaneous accelerations at $t = 1$ and $t = 2$ s. (*c*) Compare the average and instantaneous quantities and in each case explain why the larger one is larger.

12. An electron, starting from rest, has an acceleration that increases linearly with time; that is, $a = kt$, in which $k = 1.50$ m/s^3. (*a*) Plot a versus t during the first 10-s interval. (*b*) From the curve of part (*a*) plot the corresponding v versus t curve and estimate the electron's velocity 5 s after the motion starts. (*c*) From the v versus t curve of part (*b*) plot the corresponding x versus t curve and estimate how far the electron moves during the first 5 s of its motion.

13. Suppose that you were called upon to give some advice to a lawyer concerning the physics involved in one of her cases. The question is whether a driver was exceeding a 30-mi/h speed limit before he made an emergency stop, brakes locked and wheels sliding. The length of skid marks on the road was 19.2 ft. The police officer made the assumption that the maximum deceleration of the car would not exceed the acceleration of a freely falling body (= 32 ft/s^2) and did not give the driver a ticket. Was the driver speeding? Explain.

14. A train started from rest and moved with constant acceleration. At one time it was traveling at 33.0 m/s, and 160 m farther on it was traveling at 54.0 m/s. Calculate (*a*) the acceleration, (*b*) the time required to travel the 160 m, (*c*) the time required to attain the speed of 33.0 m/s, and (*d*) the distance moved from rest to the time the train had a speed of 33.0 m/s.

15. When a driver brings a car to a stop by braking as hard as possible, the stopping distance can be regarded as the sum of a "reaction distance," which is initial speed times reaction time, and "braking distance," which is the distance covered during braking. The following table gives typical values:

Initial Speed (m/s)	Reaction Distance (m)	Braking Distance (m)	Stopping Distance (m)
10	7.5	5.0	12.5
20	15	20	35
30	22.5	45	67.5

(*a*) What reaction time is the driver assumed to have? (*b*) What is the car's stopping distance if the initial speed is 25 m/s?

16. At the instant the traffic light turns green, an automobile starts with a constant acceleration of 2.2 m/s^2. At the same instant a truck, traveling with a constant speed of 9.5 m/s, overtakes and passes the automobile. (*a*) How far beyond the starting point will the automobile overtake the truck? (*b*) How fast will the car be traveling at that instant? (It is instructive to plot a qualitative graph of x versus t for each vehicle.)

17. A sprinter, in the 100-m dash, accelerates from rest to a top speed with a (constant) acceleration of 2.80 m/s^2 and maintains the top speed to the end of the dash. (*a*) What time elapsed and (*b*) what distance did the sprinter cover during the acceleration phase if the total time taken in the dash was 12.2 s?

18. A ball is tossed vertically into the air with an initial speed somewhere between $(25 - \epsilon)$ m/s and $(25 + \epsilon)$ m/s, where ϵ is a small number compared to 25. The total time of flight for the ball to return to the ground will be somewhere between $t - \tau$ and $t + \tau$. Find t and τ.

19. Figure 2-42 shows a simple device for measuring your reaction time. It consists of a strip of cardboard marked with a scale and two large dots. A friend holds the strip vertically with his thumb and forefinger at the upper dot and you position your thumb and forefinger at the lower dot, being careful not to touch the strip. Your friend releases the strip, and you try to pinch it as soon as possible after you see it begin to fall. The mark at the place where you pinch the strip gives your reaction time. How far from the lower dot should you place the 50-, 100-, 150-, 200-, and 250-ms marks?

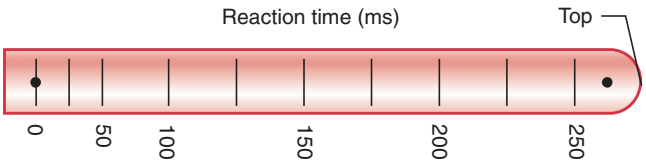

FIGURE 2-42. Problem 19.

20. While thinking of Isaac Newton, a person standing on a bridge overlooking a highway inadvertently drops an apple over the railing just as the front end of a truck passes directly below the railing. If the vehicle is moving at 55 km/h (= 34 mi/h) and is 12 m (= 39 ft) long, how far above the truck must the railing be if the apple just misses hitting the rear end of the truck?

21. A rocket is fired vertically and ascends with a constant vertical acceleration of 20 m/s^2 for 1.0 min. Its fuel is then all used

and it continues as a free-fall particle. (*a*) What is the maximum altitude reached? (*b*) What is the total time elapsed from takeoff until the rocket strikes the Earth? (Ignore the variation of g with altitude).

22. A basketball player, about to "dunk" the ball, jumps 76 cm vertically. How much time does the player spend (*a*) in the top 15 cm of this jump and (*b*) in the bottom 15 cm? Does this help explain why such players seem to hang in the air at the tops of their jumps? See Fig. 2-43.

FIGURE 2-43. Problem 22.

23. A stone is thrown vertically upward. On its way up it passes point A with speed v, and point B, 3.00 m higher than A, with speed $v/2$. Calculate (*a*) the speed v and (*b*) the maximum height reached by the stone above point B.

24. The Zero Gravity Research Facility at the NASA Lewis Research Center includes a 145-m drop tower. This is an evacuated vertical tower through which, among other possibilities, a 1-m-diameter sphere containing an experimental package can be dropped. (*a*) For how long is the experimental package in free fall? (*b*) What is its speed at the bottom of the tower? (*c*) At the bottom of the tower, the sphere experiences an average acceleration of $25g$ as its speed is reduced to zero. Through what distance does it travel in coming to rest?

25. A woman fell 144 ft from the top of a building, landing on the top of a metal ventilator box, which she crushed to a depth of 18 in. She survived without serious injury. What acceleration (assumed uniform) did she experience during the collision? Express your answer in terms of g.

26. A certain computer hard disk drive is rated to withstand an acceleration of $100g$ without damage. Assuming the drive decelerates through a distance of 2 mm when it hits the ground, from how high can you drop the drive without ruining it?

27. As Fig. 2-44 shows, Clara jumps from a bridge, followed closely by Jim. How long did Jim wait after Clara jumped? Assume that Jim is 170 cm tall and that the jumping-off level is at the top of the figure. Make scale measurements directly on the figure.

FIGURE 2-44. Problem 27.

28. A parachutist after bailing out falls 52.0 m without friction. When the parachute opens, she decelerates at 2.10 m/s^2 and reaches the ground with a speed of 2.90 m/s. (*a*) How long is the parachutist in the air? (*b*) At what height did the fall begin?

29. A steel ball bearing is dropped from the roof of a building (the initial velocity of the ball is zero). An observer standing in front of a window 120 cm high notes that the ball takes 0.125 s to fall from the top to the bottom of the window. The ball bearing continues to fall, makes a completely elastic collision with a horizontal sidewalk, and reappears at the bottom of the window 2.0 s after passing it on the way down. How tall is the building? (The ball will have the same speed at a point going up as it had going down after a completely elastic collision.)

30. A juggler juggles 5 balls with two hands. Each ball rises 2 meters above her hands. Approximately how many times per minute does each hand toss a ball?

31. What is a reasonable estimate for the maximum number of objects a juggler can juggle with two hands if the height to which the objects are tossed above the hands is h?

32. Assume that Galileo had attempted to drop two objects off of the Tower of Pisa. (*a*) If he had released the objects from his hands, but dropped one slightly sooner than the second with a time difference of $\Delta t = 0.1$ s, then what would be the vertical separation of the two objects just before hitting the ground? (*b*) What release accuracy Δt would he need so that the two

objects would have a vertical separation of less than 1 cm just prior to hitting the ground? (Ignore any effects of air friction.)

33. At the National Physical Laboratory in England (the British equivalent of our National Institute of Standards and Technology), a measurement of the acceleration g was made by throwing a glass ball straight up in an evacuated tube and letting it return, as in Fig. 2-45. Let Δt_L be the time interval between the two passages across the lower level, Δt_U the time interval between the two passages across the upper level, and H the distance between the two levels. Show that

$$g = \frac{8H}{(\Delta t_L)^2 - (\Delta t_U)^2}.$$

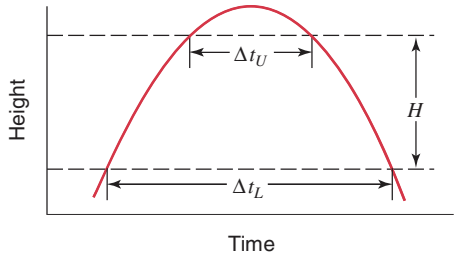

FIGURE 2-45. Problem 33.

COMPUTER PROBLEM

1. The velocity of an object is given by

$$v_x(t) = e^{-t^2/100}(t + 10 \sin \pi t).$$

Both v_x and x are zero when $t = 0$. (*a*) Numerically find the first time when v_x equals zero after the start and find the distance traveled from the origin. (*b*) Find the final position of the object as $t \rightarrow \infty$.